Aggregating the Effect of Bank Credit Supply Shocks on Firms

Juan Herreño*

UC San Diego

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Abstract

A body of compelling evidence documents that after a bank cuts its credit supply to its corporate customers, it causes a relative decline in employment and credit in those firms. I make clear what these cross-sectional effects imply for the aggregate economy. I aggregate these relative effects using a new model consistent with estimates of cross-sectional effects at different aggregation levels that allows for the central general equilibrium effects cross-sectional regressions difference out. Cross-sectional employment and credit elasticities are informative but are not enough to obtain the aggregate effects of a credit supply shock. Regional and spillover estimates impose discipline on the strength of general equilibrium effects. After imposing structure on the extent of reallocation of inputs, demand, and funds, I estimate large aggregate effects of credit supply shocks on aggregate output. General equilibrium forces moderately dampen the size of the cross-sectional effects.

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Introduction

A large cross-sectional literature has documented that a relative shock to the balance sheets of a subset of banks causes relative employment losses at firms with pre-existing relationships to the shocked banks. These research designs provide compelling evidence that bank health affects firms and local economies (Rosengren and Peek, 2000; Khwaja and Mian, 2008; Chodorow-Reich, 2014; Huber, 2018). Due to the nature of their respective research designs, these cross-sectional elasticities identify relative rather than aggregate effects, either within firms, across firms, or across regions. The consequences of the underlying shock on firms with ex-ante healthy lenders and those healthy lenders themselves are absorbed by a time-fixed effect, creating a missing intercept problem.

What can these cross-sectional estimates tell us about the aggregate effects of overall bank funding shocks on the capacity of firms to produce? The approach that I follow in this paper is to translate the relative cross-sectional impact of idiosyncratic shocks to a handful of lenders into the aggregate effects of symmetric shocks that affect every lender using a model. I run the same regressions in the model than in the cross-sectional literature to discipline structural parameters dictating the size of the relative effects and use the model to back out the size of the missing intercept.

General equilibrium forces may dampen or amplify the cross-sectional effects of credit supply shocks. Dampening forces occur because a shock that increases the cost of funding and the marginal costs to some firms will cause inputs and demand to reallocate from firms with unhealthy lenders to those with healthy lenders, expanding the scale of unaffected firms. This reallocation is mediated by a decrease in input prices and a change in relative output prices. The extent of dampening is dominated by the structure of the markets for final goods, and the market for inputs. Amplification occurs because negative funding shocks may reduce the supply of inputs or contract demand at firms not directly exposed to the shock.

The missing intercept problem in the setting of financial shocks has important consequences for empirical economists. Financial shocks affect regional input prices, like the local real wage, and key national prices, like the real interest rate. Therefore, even studies that exploit variation at the regional level face a missing intercept problem. However, papers that estimate regional effects or spillover effects within a region (Rosengren and Peek, 2000; Huber, 2018) provide crucial information for the aggregation exercise. These effects are inclusive of general equilibrium effects that operate within a region, so they impose constraints on the strength of these effects, and the remaining missing intercept problem applies to prices, like national real interest rates, that affect every region.

The main innovation in the model is its flexibility to capture the main mechanisms highlighted in the cross-sectional literature, making it a natural laboratory to answer the aggregation question. As in cross-sectional research designs, firms may borrow from multiple banks and use different forms of external finance. This paper presents the first macroeconomic model able to capture all these patterns simultaneously and answer a macroeconomic question. The extent of substitutability across banks and between forms of external finance may be imperfect, and those two elasticities of substitution are two key parameters in the model. Banks are large and internalize their market power when they price their loans and raise funding.

I discipline the elasticities of substitution of credit between banks and between sources of finance using two cross-sectional regressions that are the best practice in the empirical assessment of the firm-level effects of bank shocks. These regressions estimate the differential impact of a bank shock on credit and employment firms with stronger ties to the affected bank. I show that large frictions to reallocate inputs or demand across firms are necessary to rationalize large regional effects as documented by Huber (2018). The choice of Huber (2018) as the target for the calibration exercise, is the availability of firm-level credit and employment effects, as well as regional employment effects, and within-region spillover effects, which are crucially informative for the aggregation.

I show that cross-sectional regressions of firm-level employment growth and credit growth on exposure to granular bank-lending shocks are informative about the aggregate effects of symmetric shocks. These regressions contain information on the extent of bank credit substitutability, and the strength of this margin of substitution is relevant for aggregate output after symmetric shocks that affect all the banks and granular shocks that affect only a subset of lenders. Bank-credit growth regressions with firm fixed effects are interesting in their own right, not just as a robustness exercise. Although the elasticity of substitution across banks does not affect aggregate output after symmetric shocks that affect every bank, it is relevant for the cross-sectional and aggregate consequences of the granular shocks affecting a subset of banks for which we have good causal effects estimates. So, the firm fixed-effect estimator provides useful information when moving from the cross-sectional effects of granular shocks to the aggregate effects of symmetric shocks.

Firm-level credit and employment regressions suffer from an observational equivalence problem. It is not possible to distinguish economies with different degrees of financial frictions by using these firm-level regressions exclusively. The reason is that although cross-sectional elasticities of output are decreasing in the ability of firms to substitute between financial sources, they are increasing in the elasticity with which inputs and demand reallocate across firms. Only after pinning down the strength of the reallocation of demand and inputs, it is possible to use the regressions to uncover the substitutability of finance sources.

One key result of the paper is to guide applied researchers on the multiplier to apply to their cross-sectional estimates in order to recover a lower bound of the aggregate results and to recover the structural parameters that map into the extent of insubstitutabilities of sources of finance. In order to do that, I present a simple model in which I offer guidance on the relative size of an across-firm back-of-the-envelope aggregation with respect to the aggregate effect of a counterfactual exercise where all banks receive negative funding shocks. I show that second-order effects that capture the elasticity at which firms switch banks and forms of external finance are crucial. This is in contrast with earlier work by Chodorow-Reich (2014) on which I build on. In this earlier work the ratio between cross-sectional and aggregate effects depends only on the structure of product and input markets. The second-order approximation nests the first-order approximation in the limit where firms are perfectly inelastic in switching borrowers or sources of external finance, or in the limit where the funding shock affecting the economy is sufficiently small.

I extend the model to allow for spillovers of funding shocks across banks driven by deposit competition, which will amplify the cross-sectional effects in general equilibrium, and use the extended model to recover the key elasticities of substitution numerically. I focus on this general equilibrium mechanism to highlight the importance of the dynamics of the real interest rate in creating a missing-intercept problem for firm- and regional-level estimates. The idea behind the identification is the following. After a bank funding shock, firms that can replace funding from the affected bank with financing from other banks will experience little change in their credit or output due to the shock. However, if firms cannot substitute across banks but can avoid bank credit altogether, they will take on much less bank credit, but their output losses will be small. This tension implies that elasticities of firm credit and employment identify the two key parameters in the model. I target the regression coefficients estimated by Huber (2018), but the methodology can be adapted to target estimates in other settings.

I estimate an elasticity of output to lending supply of 0.2. This number means that an aggregate bank funding shock that triggers a 1 percent drop in aggregate lending causes a decline in aggregate output of 0.2 percent. Under an alternative recalibration of the elasticities of credit demand in an economy with an infinite elasticity of reallocating inputs across firms that targets the same observed cross-sectional estimates, the aggregate effects of the same shock are three times smaller, highlighting the quantitative relevance of the observational equivalence problem I presented before. The reason is that more flexible input markets imply larger cross-sectional elasticities after the same shock. To target the same cross-sectional patterns, frictions in the financial sector must be smaller when input markets are more elastic.

Finally, I compare the magnitude of the elasticity I obtain in general equilibrium with the (partial equilibrium) back-of-the-envelope calculation. Back-of-the-envelope calculations add up the differences in every firm's outcome with respect to that of a control firm with zero direct exposure to the shock. Under my benchmark parameterization, the general equilibrium output drop is 70% the partial equilibrium one. Under an alternative parametrization with perfectly elastic input markets, the general equilibrium effects are five times smaller than in partial equilibrium. Overall, the interpretation of the evidence through the lens of the model implies that cross-sectional effects survive aggregation. The results in the model should be interpreted as a lower bound of the aggregate effects. The evidence in Huber (2018) predicts larger spillovers than my model, which would increase the aggregate elasticity.

Literature Review To measure the effects of an aggregate lending cut on aggregate output, I rely on a large and robust empirical literature that inquires about the effects of bank health in a cross- section of firms and banks. This cross-sectional literature exploits variation in bank exposure to funding shocks and variation in the exposure of firms and regions to different banks. This body of evidence concludes that bank disruptions affect the allocation of firm credit, as in Khwaja and Mian (2008), Gan (2007), Schnabl (2012); Iyer, Peydro, da Rocha Lopes, and Schoar (2013) among others; firm outcomes like employment and sales, presumably because of the existence of sticky firm-bank relationships, as in Chodorow-Reich (2014); regional outcomes, as in the seminal work by Rosengren and Peek (2000), Ashcraft (2005), Greenwood, Mas, and Nguyen (2014), and Huber (2018); and across countries, as in Biermann and Huber (2019), and Xu (2020).

This paper contributes to the broad literature that uses cross-sectional estimates to investigate the macroeconomic effects of macro and micro shocks. The approach I follow in this study uses causal effects measured in cross-sectional settings as inputs to measure an aggregate elasticity, in this case the elasticity of aggregate output to aggregate lending shocks. Nakamura and Steinsson (2017) survey the literature and discuss its challenges. Particularly relevant is the work of Catherine, Chaney, Huang, Sraer, and Thesmar (2022)studying the aggregate effects of collateral constraints, Mian et al. (2022), who device a model-free approach to aggregate individual effects into local general equilibrium effects, Huber (2023), who offers guidance on how to directly estimate spillovers using quasi-experimental variation, and Sraer and Thesmar (2018) on how to aggregate quasi-experimental evidence to estimate misallocation. Compared to this line of work, this paper is able to compute counterfactuals to extrapolate the differential effects of idiosyncratic shocks into the aggregate effects of a counterfactual where all the banks are shocked. Moreover, this approach is complementary to Mian et al. (2022), contributing with a framework to aggregate local aggregate effects into national aggregate effects.

Methodologically, this paper builds on pieces of work that embed banks with market power on lending and deposits to understand macroeconomic outcomes. Examples are Ulate (2021) and Abadi et al. (2023) and Balloch and Koby (2019), among others. In this paper banks market power emerges from the specificity of its relationship with its customers. I microfound this outcome from a series of discrete choice problems at the firm level that give as an outcome a credit demand function. In this sense, this approach is similar to the Ricardian models in the Eaton and Kortum (2002) spirit, used to characterize trade flows between countries. Instead, however, I use it to characterize the flow of credit from banks to firms and firms' decisions about how much to borrow. In particular, I use a nested Frechet distribution, which allows for additional flexibility in the patterns of substitutability within a nest and across nests. Examples are Zárate (2022) and Galle et al. (2022), among others.

1 Static Model

In this section I present a model that is flexible to incorporate the credit and output effects observed in the cross-section after a granular credit supply shock, and use it to analyze the effects of bank health on aggregate output. The model features a continuum of firms, a discrete number of bank types, and a representative household. Firms borrow from multiple banks simultaneously. Banking relationships are imperfectly substitutable in the sense that the relative demand for funding from a particular bank is downward sloping, not horizontal. Bond financing is an imperfect substitute for bank credit. This model is static and makes a number of simplifications that I will relax later in the paper.

1.1 Firms

There is a continuum of monopolistic competitive firms producing differentiated varieties. Firm are indexed by j in the unit interval. The demand schedule for each firm is given by:

$$y_j = Y p_j^{-\eta}, \tag{1}$$

where p_j is the relative price of variety j, y_j is the quantity demanded of each variety and y is aggregate demand. The aggregate price level is the numeraire.

Each variety is produced by mixing a continuum of intermediates indexed by ω. The firm aggregates the intermediates via a CES function with elasticity of substitution σ.

$$y_j = \left(\int_0^1 \left(y_j(\omega)\right)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}. \tag{2}$$

Each intermediate good ω is produced linearly with labor, and firms receive a productivity shifter z

$$y_j(\omega) = z_j l_j(\omega). \tag{3}$$

1.2 Financing

For a given intermediate, firms decide whether to issue a bond or look for funding from a bank. Firms that choose bank financing must select an individual bank to finance each intermediate. Different financing options may in principle offer different terms, and firms face shifters that reflect the comparative advantage of financing an intermediates with a given financing option, reflecting bank specialization as in Paravisini et al. (2023) or differential asymmetries of information.

Because firms need to finance a continuum of intermediates, the cost of funds for the firm, which shapes its marginal cost, does not depend on the realization of the financing cost of any particular intermediate good, but on structural parameters dictating the distribution of the shifters, and the interest rates exclusive of the comparative advantage shocks.

1.2.1 Choice of a financing source

The total cost of financing intermediate ω is given by $TC_j(\omega)$, which consists of the wage bill and the financing costs of financing the wage bill, which after replacing the linear production function is given by

$$TC_j(\omega) = \frac{w_j}{z_j} R_j(\omega) y_j(\omega), \tag{4}$$

where $R_j(\omega)$ is the effective gross interest rate firm j must pay to finance the expenditures of intermediate ω. As part of its cost minimization problem, firm j looks for the cheapest financing option.

In particular, at the intermediate level, the firm chooses the financing option that minimizes the cost of financing that intermediate input, achieving a cost of financing for that intermediate ω of

$$R_j(\omega) = \min\left\{\min_{b\in\mathcal{B}}\left\{\frac{R_b}{\epsilon_{jb}(\omega)}\right\}, \frac{R_{\mathcal{M}}}{\epsilon_{j\mathcal{M}}(\omega)}\right\}. \tag{5}$$

Here $\mathcal{B}$ and $\mathcal{M}$ denote a set of banks and an a market option ($N_{\mathcal{M}} = 1$ without loss), b indexes one out of the $N_{\mathcal{B}}$ banks in the economy. The effective cost the firm perceives if it were to choose a financing option is equal to the cost of funds of that option, over a shifter, that captures all the idiosyncratic reasons why one option may be better for some intermediates than others. For example, some projects of the firm may entail differential verification costs (Townsend, 1979) across lenders, or simply some banks may have a comparative advantage on some segments driven by their historical expertise or geographical know-how (Paravisini et al., 2023).

I assume the vector $\boldsymbol{\epsilon}$, which collects the shifters for the banks and the bond option is drawn from a nested Fréchet Distribution with CDF

$$F_j(\boldsymbol{\epsilon}) = \exp\left\{-\sum_{s\in(\mathcal{B},\mathcal{S})}\psi_s\left(\sum_{b=1}^{N_s} T_{jb}\epsilon_{sb}^{-\theta}\right)^{\frac{\varphi}{\theta}}\right\}.$$

This distribution has been used by Dingel, Meng, and Hsiang (2019), Lashkaripour and Lugovskyy (2018), Galle et al. (2022), Zárate (2022), and it extends the Fréchet distribution common in the Ricardian model of international trade of Eaton and Kortum (2002). The nested Fréchet distribution captures the variation in the advantage of financing a given intermediate input both across banks (some banks are better than others) and across financing options (some intermediates goods are better suited to be financed with bank credit).

The T_{jb} parameters capture the strength of the long-term banking relationship between firm j and bank b, or the absolute advantage of bank b in providing funding for firm j. Whenever $\varphi = \theta$, firms substitute between a given bank credit and bond financing with the same elasticity as they substitute across banks.

Under the assumptions stated before, we can characterize the share of expenditures financed with each bank conditional on choosing bank financing, ν_{jb}, and the cost of bank credit for the firm, R_{jB}. Formally,

$$\nu_{jb} = \frac{T_{jb} R_b^{-\theta}}{\sum_k T_{jk} R_k^{-\theta}}. \tag{6}$$

$$R_{jB} = \left(\sum_{b \in \mathcal{B}} T_{jb} R_b^{-\theta} \right)^{-1/\theta}, \tag{7}$$

The borrowing shares depend on θ, the dispersion of the shifters across banks, which plays the role of an elasticity of substitution of funding across banks, and on T_{jb}, which is the relative strength of the banking relationship between firm i and bank b. The share of expenditures financed with the banking sector s_j, is given by

$$s_j = \frac{\psi_B R_{jB}^{-\varphi}}{\psi_B R_{jB}^{-\varphi} + (1 - \psi_B) R_{jM}^{-\varphi}}, \tag{8}$$

where I have normalized $\psi_B + \psi_M = 1$, and the effective cost of funds for the firm is given by the cost of funds index R_j

$$R_j = \left(\psi_B R_{jB}^{-\varphi} + (1 - \psi_B) R_{jM}^{-\varphi} \right)^{-1/\varphi}. \tag{9}$$

This discrete choice block is a microfoundation of the desired financial structure of the firm. When bank credit becomes more expensive ($R_{jtB} \uparrow$), the firm moves away from bank lending ($s_{jt} \downarrow$). The elasticity at which the substitution occurs is given by φ. Adding additional additional layers of financing options is not complicated, but I keep the model stylized in order to take it to the data.

Figure (1) plots the share of financing from the banking sector as a function of the cost of bank funds for different values of φ. The figure shows that as φ increases, the relative demand schedule for bank funds becomes more elastic. In the limit, when $\varphi \to \infty$ the relative demand curve becomes horizontal, and firms are perfectly elastic in switching between bank funding and self-finance. On the other side, when φ becomes smaller, the

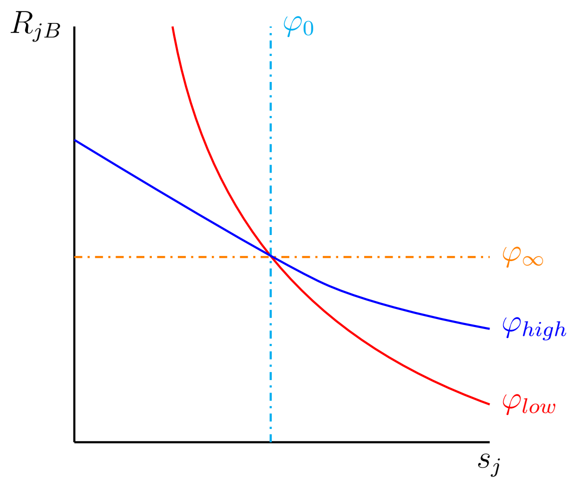

Figure 1: Relative demand schedule for bank credit

Note: The figure shows the ratio of bank loans to financing needs of the firm s_j as a function of the effective cost of bank loans R_{jB} for several values of the elasticity of substitution between bank loans and self-finance φ.

share of bank financing is less sensitive to its cost.

When θ is higher, the demand curves for funding for a particular bank become flatter, which I show in Figure (2). In the limit, when $\theta \to \infty$ the demand curve becomes horizontal, and firms are perfectly elastic in switching between banks. On the other side, when θ tends to zero, the share of bank financing from bank b is less sensitive to bank b's lending rate.

1.3 Workers

There is a representative household that we keep as simple as possible. It consumes (C) and supplies labor L. The household maximizes the utility function, which takes a GHH form,

$$U(C_t, L_t) = \frac{1}{1-\gamma}\left(C_t - \frac{L_t^{\phi+1}}{1+\phi}\right)^{1-\gamma} \tag{10}$$

Where L_t is an aggregator of the labor supply to different firms in the economy:

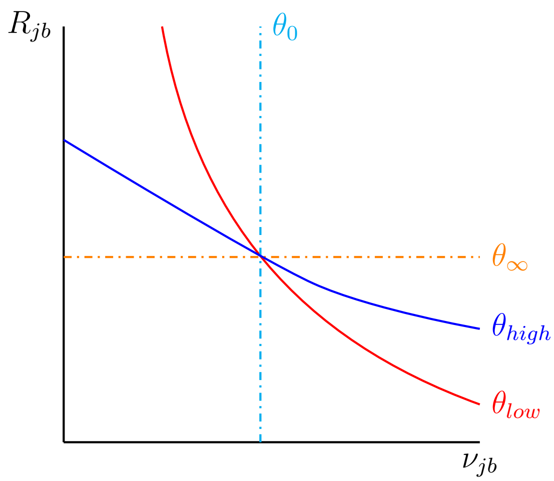

Figure 2: Relative demand schedule for bank-specific credit

Note: The figure shows the ratio of bank b credit to total bank credit chosen by firm j, ν_{jb}, as a function of the effective cost of bank loans from bank b R_{jb} for several values of the elasticity of substitution between banks θ.

$$L_t = \left(\int L_{jt}^{\frac{1+\alpha}{\alpha}} dj \right)^{\frac{\alpha}{1+\alpha}}. \tag{11}$$

Workers maximize utility subject to a budget constraint $\int w_{jt} L_{jt} dj + \Pi_j = C_t$, where Π_j are aggregate profits of the firms in the economy. Households supply labor according to the following relationship:

$$L_t = w_t^{1/\phi}, \tag{12}$$

$$L_{jt} = L_t \left(\frac{w_{jt}}{w_t} \right)^{\alpha}, \tag{13}$$

where w_t is defined as L_t^{ϕ}. Workers demand higher pay in order to work more hours at the same firm. When $\alpha \to \infty$, the labor market operates under a single wage rate $w_{jt} = w_t$ $\forall j$.

1.4 Discussion of Assumptions

In this model, it is assumed that lending rates are exogenous. Later in the full model, I will specify the bank problem that gives rise to the lending rates in equilibrium as a function of the market structure and the ease of securing funding. I also assumed that the profits belong to the workers. The problem of the firm owners is included in the full model as well.

I assume that the substitutability across banks, and across forms of finance are different economic objects, so they can take different values. This assumption is taken in line with the corporate finance literature that points that local bank finance is difficult to substitute (Paravisini, 2008). The alternative would be to assume that substitution between banks and between forms of external finance is governed by the same parameter, which would simplify the identification argument.

Second, I assumed that the main margins of reallocation in the model are reallocation of demand and inputs, assumptions I make so the starting point of this model is the model of Chodorow-Reich (2014). I assumed that the interaction between firms are limited to competition for demand and inputs. Importantly, I exclude the possibility of input output networks, or agglomeration externalities as in Huber (2018), which would increase the aggregate effects for a given size of a cross-sectional elasticity. I make that decision with the idea of providing a lower bound for the aggregation exercise. I also leave aside the effects of bank shocks on firm entry and exit. However the full model will capture in a stylized way that the interaction of firms for demand and inputs, and the interaction of banks competing for deposits and allocating loans as the first order factors in aggregating up the cross-sectional elasticities in this literature.

In the limit in which firms are fully unable to substitute across banks or across external financing alternatives, this model collapses to the model in the online appendix of Chodorow-Reich (2014). Later in the full numerical model, I introduce competition across banks, which is another layer in which this model differs from that key reference in the literature.

2 Characterization

The focus of this section is to characterize the elasticity of aggregate output to an exogenous hike in the cost of funds of a particular bank, and to a similar shock that affects the whole banking sector symmetrically. I focus on the aggregation problem for the case in which the no-sorting identifying assumption of the cross-sectional regression holds exactly in the economy.

In this section I present two main results. First, aggregate and cross-sectional effects on output of a rate hike of an individual bank are different, and it is a priori unclear which of them is larger. The difference in magnitude is dominated by the difference in the Frisch elasticity of the labor supply and the easiness to reallocate demand and inputs across firms. When it is easy to reallocate labor and demand across firms, then up to a second order the cross-sectional effects of output are larger.

Second, although greater frictions in the banking sector, in the form of low elasticities of substitution of funds between banks and between funding alternatives increase the output losses caused by lending rate hikes, it is not possible to back out from a single cross-sectional elasticity the structural parameters that determine the response of aggregate output.

2.1 The Aggregate Effects of Loan Term Changes in One Bank

I take the validity of the research designs in the cross-sectional literature at face value. Therefore, I impose the following assumption, which implies there is no sorting of banking relations, or alternatively, that an empirical researcher has an instrument in order to deal with endogenous sorting.

Assumption 1. *There is no sorting in financial relations. That is, firm-level productivity z_j and the strength of bank lending relationships T_{jb} are orthogonal. I rule out the possibility that banks more prone to lending rate hikes are more closely linked to firms more prone to lower productivity draws.*

The first results of this section hold under the following assumption:

Assumption 2. *Define by R an arbitrary level of interest rates charged by every bank and the bond instrument. At these rates, the level of aggregate output is defined as $\bar{Y}$. For an arbitrary bank b, the lending terms are disrupted to Re^u, for a positive and sufficiently small u.*

Proposition 1. ***The aggregate effects of granular shocks.*** *Under Assumptions (1) and (2), up to the second order, the log change of output caused by an increased in the lending cost of bank b is given by:*

$$\Delta \log Y^{id} \approx -\frac{1}{\phi}\psi_B u \left(\bar{\nu}_b - \theta\frac{u}{2}\Upsilon_1 - \varphi(1-\psi_B)\frac{u}{2}\Upsilon_2 - \psi_B\Omega\Upsilon_2\frac{u}{2}\right), \tag{14}$$

where $\Delta \log Y^{id}$ makes clear we are talking about the change in output caused by an idiosyncratic bank shock, $\bar{\nu}_b = \int_0^1 T_{jb}dj$ and σ_b^2 are the average and the variance of the market share of bank b across firms in the symmetric equilibrium; $\Upsilon_1 = \bar{\nu}_b - \sigma_b^2 - \bar{\nu}_b^2$, $\Upsilon_2 = (\sigma_b^2 + \bar{\nu}_b^2)$, and $\Omega = \frac{\eta-\alpha+\eta\alpha(1-\phi)}{\phi(\alpha+\eta)}$ are constants that capture the distribution of borrowing shares in the symmetric equilibrium and the structure of input and demand markets.

If a researcher is willing to abstract from second-order effects, which is an accurate approximation for small enough shocks, such that $u^2 \approx 0$, output losses are given by

$$\Delta \log Y^{id} \approx -\frac{1}{\phi}\psi_B\bar{\nu}_b u, \tag{15}$$

Proof: See Appendix

Proposition (1) shows that for a shock of size u to the lending terms of one bank, the response of output depends on three terms. The first term measures the direct effect of the shock up to second order, abstracting from any substitution in financing sources. The drop in output will be proportional to the relevance of the affected bank $\psi_B \bar{\nu}_b$, weighted by the Frisch elasticity of labor supply $1/\phi$. When labor supply is inelastic, the increase in the cost of funds in the aggregate will be compensated for by a fall in the aggregate wage. The second term captures a counteracting force from the ability of the firms in the economy to substitute the affected bank. Importantly, θ, the cross-bank elasticity of substitution, helps determine this second term. In a similar way, the third term captures the ability of the economy to switch from using bank credit altogether, which is determined by φ.

Proposition (2), shows that the response of output depends on observables, like the average bank-dependence of the real sector, ψ_B, the average market share of the disrupted bank, $\bar{\nu}_b$, or the dispersion of the market shares, σ_b^2. It also depends on well-studied parameters like the Frisch elasticity of labor supply (see Chetty et al. (2011)), the elasticity of substitution across goods (see Broda and Weinstein (2006)), or the firm-specific elasticity of labor supply (see Webber (2015)) . The output response also depends on two less-studied parameters: the elasticity of substitution of funding from a given bank θ, and the elasticity of substitution of bank-credit φ. In later sections of the paper I discuss the strategy I use to recover these parameters from the cross-sectional evidence and use them to estimate the effects of an aggregate bank disruption.

2.2 The Aggregate Effects of Overall Loan Term Disruptions

Now I extend the results in Proposition (1) for a generalized disruption in the loan terms of all the banks. Proposition (2) presents the main result of this section, using Assumption (3).

Assumption 3. *Assume the lending terms of all banks are disrupted from R to Re^u, for a positive and sufficiently small u. Keep the self-finance rate equal to R.*

Proposition 2. *The Aggregate Effects of Symmetric Shocks: Under Assumptions (1) and (3), up to a second order, the fall of output triggered by a symmetric increase in the lending terms of all the banks is given by:*

$$\Delta \log Y^{all} \approx -\frac{1}{\phi}\psi_B u \left(1 - \varphi(1-\psi_B)\frac{u}{2} - \Omega\psi_B \Upsilon_2 \frac{u}{2}\right), \tag{16}$$

where Υ_1, Υ_2 and Ω have the same definition as in Proposition 2.

If a researcher is willing to abstract from second-order effects, which is an accurate approximation for small enough shocks, such that $u^2 \approx 0$, output losses are given by

$$\Delta \log Y^{all} \approx -\frac{1}{\phi}\psi_B u. \tag{17}$$

where $\Delta \log Y^{all}$ makes clear that we are measuring the aggregate effects of a symmetric shock that effects every lender.

Proof: See Appendix

Proposition (2) shows that the elasticity of substitution between banks θ is irrelevant at the aggregate level after a symmetric shock. However, the elasticity of substitution away from bank lending φ is still important through its second-order effect on aggregate output. Up to a first order approximation, the response of aggregate output is determined by the market share of the banking sector on the credit demand of firms, and the Frisch elasticity of labor supply.

2.3 The Cross-Sectional Effects on Firm Production

The elasticity of firm production to a disruption in the terms of loans of bank b has been estimated in the empirical macroeconomics and corporate finance literatures through the following regression:

$$\Delta \log \mathrm{Y}_j = \beta_0 + \beta_{output} T_{jb} + \epsilon_j, \tag{18}$$

where Δ is the difference operator between a pre- and a post-period where the experiment in Assumption 2 materialize. The right hand side variable is the pre-existing exposure of firm j to bank b, measured by T_{jb}, assumed exogenous as in Assumption 1. The main empirical concern behind Assumption 1 is that banks that are more prone to receiving funding shocks are also more likely to pick bad firms, which would induce a covariance between changes in lending and changes in firm outcomes even in absence of a causal link from credit supply shocks to firms. The empirical literature has addressed that problem by using an instrumental variables (IV) approach. I take the validity of those IV research designs as given. For a discussion of sorting between firms and banks see Chang, Gomez, and Hong (2020).

The elasticity of production with respect to pre-existing exposure is characterized in Proposition (3)

Proposition 3. *Under assumptions (1) and (2), the regression coefficient of a regression of firm-level output growth on pre-existing exposures, accurate up to a second-order, is given by the following expression*

$$\beta_{output} = -\frac{\eta\alpha}{\alpha+\eta}\psi_B u\left(1+\psi_B\mathcal{T}\frac{u}{2}-\theta\left(1-\mathcal{T}\right)\frac{u}{2}-\varphi(1-\psi_B)\mathcal{T}\frac{u}{2}\right). \tag{19}$$

If the underlying shock is small enough such that $u^2 \approx 0$, then a first-order approximation is sufficient to characterize the regression coefficient as given by

$$\beta_{output} = -\frac{\eta\alpha}{\alpha+\eta}\psi_B u. \tag{20}$$

For a constant $\mathcal{T} = \left(\frac{cov(T_{jb}^2, T_{jb})}{var(T_{jb})}\right)$.

To compute a back of the envelope aggregation of the cross-sectional effects, empirical economists often compute the product of an estimate of β_{output} times the average exposure of firms to the affected bank, $\bar{\nu}_b$.

Proof: See Appendix

Proposition 3 makes clear that up to a second order, as the elasticity of substitution across banks (θ) and the elasticity of substitution away from bank credit (φ) increase, the firm-level effects of a bank disruption on output become smaller. On top of the frictions in the banking sector, the structure of the goods market (η), and the structure of labor markets α determine the cross-sectional effects of the bank disruption. When α tends to infinity, the first term in equation tends to η. When both α and η tend to infinity, the cross-sectional effects diverge; in this situation all production would take place in the firm with the lowest marginal cost.

2.4 Aggregation of Cross-Sectional Effects in the Simple Model

There are two exercises an empirical economist may be interested in pursuing that this simple model can answer. The first one is to compute the aggregate effects implied by the cross-sectional effects of an idiosyncratic bank shock on firms. The second exercise is to scale the cross-sectional effects in a counterfactual where all, and not just one bank, suffered a shock. In concise terms, two objects of interest require comparing the results in Proposition 3 to those in Proposition 1 or those in Proposition 2.

If a researcher has a back of the envelope aggregation, as presented in Proposition 3, and is willing to assume that the shock is small enough such that it is appropriate to ignore second order effects, then the ratio between the aggregate effects an idiosyncratic bank shock

$$\frac{\Delta \log Y^{id}}{\beta_{\text{output}} \times \bar{\nu}_b} \approx \frac{\alpha + \eta}{\phi \eta \alpha}, \tag{21}$$

and the ratio between the back-of-the-envelope aggregation and the aggregate effects of a counterfactual where every bank suffered a shock are given by

$$\frac{\Delta \log Y^{all}}{\beta_{\text{output}} \times \bar{\nu}_b} \approx \frac{\alpha + \eta}{\phi \eta \alpha} \frac{1}{\bar{\nu}_b}. \tag{22}$$

Equation 21 makes clear that in the case of a first-order approximation, knowledge on the structure of input and demand markets is sufficient to know the size of the multiplier.

In principle, this multiplier can be larger or smaller than one, so it is not possible to bound the cross-sectional effects. Under a standard parameterization of $\alpha = 2$, $\eta = 4$ and $\phi = 1$, in the bounds of the empirical evidence by Webber (2015), Chetty et al. (2011), Broda and Weinstein (2006) the first-order-approximation multiplier in equation 21 is 75% as large as the cross-sectional effects, and in the limit where $\alpha \to \infty$, a common assumption made in macroeconomic models, the aggregate effects are 25% as large as the cross-sectional effects. These results, are reminiscent of those in the online appendix of Chodorow-Reich (2014), although for different reasons. In the model in Chodorow-Reich (2014), firms have access to a unique funding alternative, so the elasticities of substitution at the center of the model in this paper are muted. In this paper, for a small enough shock that perturbs the economy away from a symmetric equilibrium, the substitution margins across forms of external finance have second order effects.

Equation 22 makes clear that if the first-order approximation is convenient for the case studied in a research design, extrapolating the cross-sectional effects to a situation where all banks receive a shock only requires to inflate the cross-sectional effects by the inverse of the market share of the shocked bank in the research design.

Although very appealing due to how concise these expressions are, most of the literature in macroeconomics and corporate finance exploits natural experiments that entail large shocks during periods of financial distress, so a first-order approximation may be too coarse. Taking the ratio between the aggregate output responses in Proposition 2 and the cross-sectional responses of Proposition 3 yields the following expression

$$\frac{\Delta \log Y^{id}}{\beta_{\text{output}} \times \bar{\nu}_b} \approx \frac{\alpha+\eta}{\phi\eta\alpha} \frac{\left(\bar{\nu}_b - \psi_B \Omega \Upsilon_2 \frac{u}{2} - \theta \frac{u}{2} \Upsilon_1 - \varphi(1-\psi_B)\frac{u}{2}\Upsilon_2\right)}{\left(\bar{\nu}_b + \psi_B \mathcal{T} \bar{\nu}_b \frac{u}{2} - \theta \bar{\nu}_b \left(1-\mathcal{T}\right) \frac{u}{2} - \varphi \bar{\nu}_b (1-\psi_B) \mathcal{T} \frac{u}{2}\right)}, \tag{23}$$

and extrapolating from a back of the envelope aggregation to a situation where every bank receives a shock yields

$$\frac{\Delta \log Y^{all}}{\beta_{\text{output}} \times \bar{\nu}_b} \approx \frac{\alpha+\eta}{\phi\eta\alpha} \frac{\left(1-\varphi(1-\psi_B)\frac{u}{2}-\Omega\psi_B\Upsilon_2\frac{u}{2}\right)}{\left(\bar{\nu}_b+\psi_B\mathcal{T}\bar{\nu}_b\frac{u}{2}-\theta\bar{\nu}_b\left(1-\mathcal{T}\right)\frac{u}{2}-\varphi\bar{\nu}_b(1-\psi_B)\mathcal{T}\frac{u}{2}\right)}. \tag{24}$$

Equation 24 makes clear that the cross-sectional back of the envelope aggregation can be in principle arbitrarily uninformative about the aggregate effects of an overall shock. If firms are very elastic in substituting across banks (θ is large), the cross-sectional effects of an idiosyncratic bank shock may be arbitrarily small compared to the aggregate effects, since they do not depend on θ, reflecting than in the case of an aggregate shock, bank substitution is not a relevant margin of adjustment.

On top of θ being irrelevant at the second order after a symmetric shock, in general the numerator and the denominator of the second term of equations 24 and 23 do not cancel out, they depend on slightly different moments of the distribution of market shares. This is the result of using the pre-existing share of financing as the right-hand-side variable in the cross-sectional regressions.

In the next section, I discuss two possible approaches to recover the structural parameters θ and φ from regressions estimated in the cross-sectional literature. One of them exploits firm-level regressions of credit growth, and the other alternative uses a Khwaja and Mian (2008) approach of a loan growth regression of bank-firm pairs using firm-fixed effects.

Once an empirical researcher has information about θ, φ, observable statistics of the distribution of market shares, and priors on the value of commonly studied structural parameters, it is possible to compute the aggregate effects implied by a given research design.

3 Identification

In this section I use the static model to illustrate how the patterns in the data identify θ and φ, the key parameters of the model. I use the insight in this section to estimate the full model I introduce in the following section.

3.1 The Identification Challenge

In order to aggregate the firm-level experiments into the macroeconomic effects of granular or symmetric shocks to the banking sector, we need to infer the elasticities of substitution θ and φ. In particular, for the counterfactual in which every bank suffers a liquidity shock, only information on φ is needed, but the cross-sectional responses presented before depend on θ and φ. In particular, many combinations of θ and φ produce the same cross-sectional patterns, but the aggregate effects of a symmetric shock depend on φ but not on θ.

The strategy that I follow in the paper is to complement the cross-sectional effects on output that I presented in the last section with two alternatives. One of them is to compute the cross-sectional effects of overall bank credit after a liquidity shock, and the second one, to compute the within-firm Khwaja and Mian (2008) regression.

3.2 The Elasticity of Firm Borrowing

We turn to the effects of bank disruptions on firm bank-credit. We will use the following specification:

$$\Delta \log \text{Loans}_j = \beta_0 + \beta_{credit} T_{jb} + \epsilon_j, \tag{25}$$

where Δ is the difference operator between a pre-period, which I assume to be equal to the symmetric equilibrium of the model, and post-period, when a shock of size u that increases the interest rate of bank b from R to Re^u occurs. The independent variable is the pre-existing exposure of firm j to bank b, measured by T_{jb}. Gan (2007), Khwaja and Mian (2008), Schnabl (2012), and Iyer, Peydro, da Rocha Lopes, and Schoar (2013), Huber (2018), among others are examples of this approach.

Proposition 4. *Under assumptions (1) and (2), the regression coefficient of a regression of firm-level output growth on the pre-existing exposure, accurate up to a second order, is given by*

$$\beta_{credit} = \frac{1+\alpha}{\alpha}\beta_{output} - \varphi(1-\psi_B)u\left(1+\varphi\psi_B\frac{u}{2}\mathcal{T} - \theta\frac{u}{2}\left(1-\mathcal{T}\right)\right) \tag{26}$$

For a constant $\mathcal{T} = \left(\frac{cov(T_{jb}^2, T_{jb})}{var(T_{jb})}\right)$

Proof: See Appendix

Proposition 4 shows that on top of the effect on output times a multiplier (first term), there is a first order effect of the elasticity of substitution of bank credit φ on firm credit. When firms are more elastic in substituting away from bank credit, credit falls by more. The substitutability across banks limits the fall in credit since it limits the size of both terms. The intuition is clear, when firms can more easily move across banks, they find less necessary to reduce bank credit and suffer smaller output losses, with direct effects on credit demand.

3.3 Identification Argument

The elasticity of credit becomes larger (more negative) when φ is larger and when θ is smaller. The elasticity of output becomes larger when both φ and θ are smaller. Therefore it is possible to back out the values of these two coefficients once we take a stance on the other coefficients that determine the cross-sectional elasticities.

The identification argument is represented in Figure 3. The figure presents two locus of points in the space φ - θ, which produce a given estimate for the elasticity of credit and production, after taking a stance on the other parameters of the economy.

Start by placing yourself on point b_1, in the locus of β_{credit}. Now arbitrarily increase the value of φ. Since a larger φ causes the elasticity to be larger in absolute value, in order to keep the elasticity constant we must move θ in a direction that compensates for the change in φ. That is, we need to make θ larger, making firms more elastic with respect to a given bank. This argument implies that the locus of points (φ and θ) that keeps the

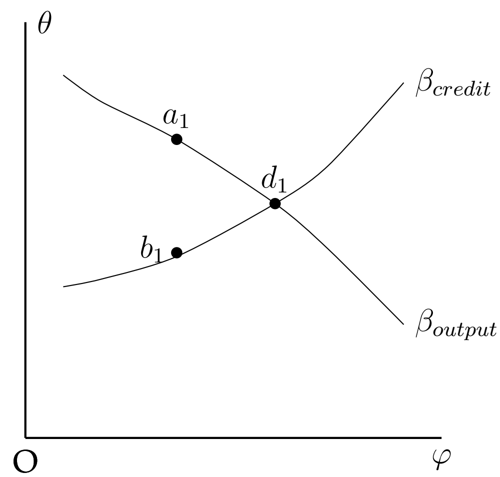

Figure 3: Identification argument for φ and θ.

Note: The figure plots the loci of points (φ and θ) that achieve a given value for β_{output} and β_{credit} after taking a stance on the other parameters. The intersection of the two loci gives the value of θ and φ.

regression coefficient β_{credit} constant is upward sloping.

Now place yourself on top of point a_1 on the locus of β_{output}. Once again move to a larger value of φ. When firms are more elastic to substitute bank credit, the elasticity of production becomes smaller in absolute value. In order to keep its value constant, we need firms to be less able to switch from the affected lender, making θ smaller. Therefore, the locus of points is downward sloping.

3.4 Firm Fixed Effects Estimator

An alternative strategy to estimate θ and φ is to use the the fixed-effect regression. In the following sections I will use the elasticities estimated by Huber (2018) to parameterize the model, which does not present firm fixed effect regressions, so I will use the argument presented before. However, in settings where the within-firm regression is available, it may be used by empirical researchers to recover the value of θ.

Firm fixed-effect regressions provide information about θ, the elasticity of substitution of funds across banks, and no information about the elasticity of substitution of bank credit (φ), the substituability of goods in the goods market (η), or the ability to reallocate labor across firms (α). The result carries significant economic content. Firm fixed-effects regressions compare firm reallocations of credit demand across banks, abstracting from any economic mechanism that operates across firms (η and α), or that does not depend on a firm-bank pair (φ).

In the previous section I showed that θ is irrelevant up to the second order to determine drops in aggregate output after an aggregate disruption of the banking sector, then the fixed-effect regression estimation is, on its own, uninformative about such aggregate experiment. However, firm fixed-effect regressions are still important. By identifying θ, they can be combined with other cross-sectional regressions to recover φ, or studying θ is interesting to determine aggregate output fluctuations after an idiosyncratic bank shock.

The within-firm fixed effect regression takes the following form

$$\Delta \log \text{Loans}_{jb} = \omega_j + \beta_{\text{fixed effect}} T_{jb} + \varepsilon_{jb} \tag{27}$$

The log of loan sizes between a bank b and firm j is given in the model by

$$\log \text{Loans}_{jb} = \log \mathcal{C} + \frac{(\eta-1)(\alpha+1)}{\alpha+\eta} \log z_j - \left(\frac{\eta(\alpha+1)}{\alpha+\eta} - \varphi\right) \log R_j - \varphi \log R_{jB} + \log \nu_{jb}, \tag{28}$$

for a constant $\mathcal{C}$ that captures aggregate and firm-level outcomes that are soaked into the constant term. Demeaning this object to compute the within-firm loan variation across banks, and computing a before-after difference, yields an expression for $\Delta \tilde{\text{Loans}}_{jb} = \Delta \log \nu_{jb} - \Delta \bar{\log} \nu_{jb}$. Up to a second-order approximation, the firm fixed-effect estimator yields

$$\beta_{\text{fixed effect}} = -\theta u + \theta^2 \frac{u^2}{2} (1 - \mathcal{T}). \tag{29}$$

Equation (29) makes clear that up to a second order, the fixed-effect estimator identifies θ, the elasticity of subsitution of funds across banks. As a consequence, a researcher can use the fixed-effect estimator, along with the employment regression in order to disentangle θ and φ separately.

3.5 Observational Equivalence

Cross-sectional estimates comparing firm effects after an idiosyncratic bank shock suffer from an observational equivalence problem. Equation (2.3) makes clear that the firm-level output effects of a lending cut depend on two distinct terms. The first one $\frac{\eta\alpha}{\eta+\alpha}$ captures the structure of labor and goods markets, while the second term $\left(1-\theta\frac{u}{2}\mathcal{T}-\varphi(1-\bar{s})\frac{u}{2}.\mathcal{T}\right)$ captures the structure of financial markets. Note that the first term is present in previous studies, like the online appendix of Chodorow-Reich (2014). The second term is new to this study. The presence of both terms is the source of the observational equivalence problem.

The cross-sectional elasticity of output would be indistinguishable for an econometrician in worlds with distinct insubstitutabilities of financial sources. The elasticity could be similar in an economy where goods and labor can move easily across firms (η and α large) and financial sources are very substitutable (θ and φ large), and another world where the reverse happens. A priori, it is not possible to assert insubstitutabilities in finance from a large cross-sectional estimate, nor reject them from small elasticities.

In order to use the cross-sectional elasticities to learn about the extent of financial frictions once we pin down the elasticities that govern the ability of the economy to reshuffle demand and input after a shock that affects a subset of the firms. The mechanisms that induce observational equivalence in the model are well-studied objects in economics, so using the data plus the model the observational equivalence problem can be solved, this is one of the main contributions of this paper.

4 Full Model

In this section I embed the simple model in a consumption/savings general equilibrium model in order to make the total amount of deposits endogenous, and let banks set lending rates as a response to balance sheet disruptions. The basics of the model are the same as in the simple model, and here I only present the new blocks of the model.

Time is continuous. Space is contained in a $[0, 1]$ interval. $N_{\mathcal{B}}$ banks are uniformly spaced in this interval. Firms are distributed uniformly over space. I take as primitive of the model the closeness of firm j to bank b, and denote it by T_{jb} as in the simple model. T_j, a vector of size $N_{\mathcal{B}} \times 1$ specifies the closeness of firm j with each bank, and given by $T_{j,b} = \max\{1 - \bar{d} \times d_{j,b}, 0\}$ where $d_{j,b}$ is the distance between firm j and bank b, and $\bar{d}$ is a constant that determines how the distance between a firm-bank pair affects the ease of creating banking relationships. In the extreme where $\bar{d} = 0$, firms are equally likely to borrow from banks regardless of their distance. When $\bar{d}$ increases, firms only use banks that are close to them.

Each firm is owned by an entrepreneur, with utility function $u(c_{it}) = \frac{c_{it}^{1-\gamma}}{1-\gamma}$. Each entrepreneur solves the following problem:

$$\max_{c} \mathbb{E}_0 \int_0^\infty e^{-\rho t} u(c_{it}) dt.$$

They maximize utility subject to the budget constraint:

$$\dot{a}_{it} = r_{it}^d a_{it} + \pi_{i,t}^* - c_{it}$$

That is, entrepreneurs earn interest income at rate r_{it}^d on their wealth a_{it}, earn profits $\pi_{i,t}^*$, and consume c_{it}. The effective rate of deposits r_{it}^d is a weighted average of the deposit rates at different banks $r_{dt} = \sum_k \omega_{kt} r_{dkt}$. And weights given by $\omega_{bt} = \frac{R_{bdt}^X}{\sum_k R_{kdt}^X}$. This functional form for the deposit shares is symmetric with the way that firms allocate their loan demand across banks and may be derived from a discrete choice problem, which I do not expand on for brevity. Profits are given by $P_{jt}Y_{jt} - w_{jt}L_{jt}R_{jt}$, prices are given by $\frac{\eta}{\eta-1}MC_{jt}$, and marginal costs are given by $MC_{jt} = \frac{w_{jt}}{z_{jt}}R_{jt}$.

4.1 Banks

Banks compete by setting rates. Banks understand the structure of demand of each firm, but do not internalize the aggregate consequences of their actions. That is, banks take the aggregate wage and aggregate output as given, but they understand that firms can substitute towards other banks, or substitute away from bank credit, and that firm optimal scale is decreasing in its relative price. I allow for banks to price-discriminate across firms.

The profits that bank b gets from its relationship with firm j are given by

$$\Pi_{jb} = w_j L_j s_j \nu_{jb}(R_b - R_{bd}).$$

I am saving on the notation by eliminating the time subscript. The condition

$$R_{bj} = R_{bd}\frac{\tilde{\theta}_{jb}}{\tilde{\theta}_{jb} - 1}$$

For $\tilde{\theta}_{jb} = \theta + \nu_{jb}(\varphi - \theta) + \left(\eta\frac{1+\alpha}{\alpha+\eta} - \varphi\right)\nu_{jb}s_j$, characterizes the optimal pricing of the loans for each bank.

A bank with zero mass ($\nu \to 0$) faces an elasticity of substitution θ, the elasticity at which firms switch banks. A monopolist bank ($\nu \to 1$) lending to a firm that are fully dependent on bank credit ($s \to 1$), faces an elasticity of substitution $\eta\frac{1+\alpha}{\alpha+\eta}$, the elasticity at which higher costs translate into lower firm scale and correspondingly to lower loan demand. The elasticity is positive since φ,η, and θ are positive, and ν_{bj} and s_j are between zero and one. Banks charge variable markups. This is an important departure from models with constant elasticities of substitution.

The balance sheet of the bank is given by:[1]

$$\text{Loans}_{bt} = \text{Deposits}_{bt} + \text{Equity}_{bt}, \tag{30}$$

where the loans granted by a bank are the integral of the loans given to each firm in the economy, given by

$$\text{Loans}_{bt} = \int_0^1 \text{Loans}_{jbt} dj = \int_0^1 s_{jt} w_{jt} L_{jt} \nu_{bjt} dj. \tag{31}$$

Similarly, deposits are equal to the integral of the deposits that the bank gets from all entrepreneurs in the economy

$$\text{Deposits}_{bt} = \int_0^1 \text{Deposits}_{jbt} dj = \int_0^1 \omega_{bt} a_{jt} dj. \tag{32}$$

I assume that Equity_{bt} is exogenous, and that banks are owned by agents outside the economy. It is simple to change that assumption on the ownership of the banking sector.

The supply of deposits at a given bank depends positively on its deposit rate, while the demand for loans depends negatively on it, through its negative relationship with the lending rate and the positive relationship between lending and deposit rates. Therefore, after a decrease in the right-hand side of the balance sheet, the bank will respond by increasing the deposit and lending rates accordingly, balancing out its balance sheet again.

The aggregate state vector is $\mathbb{S} = (\textbf{Equity}, X)$, where $\textbf{Equity}$ is a $K \times 1$ vector of the equity of each of the K banks in the economy, and X is the distribution of entrepreneurs over their individual state-space $(z, a, \mathbf{T})$.

1. Entrepreneur's optimization. Taking $w(\mathbb{S})$, $R_k(\mathbb{S})$, $R_k^d(\mathbb{S})$ as given, entrepreneurs maximize utility and their firms maximize profits.

2. Household problem. Taking $w(\mathbb{S})$ as given, households maximize utility

[1] In the model, I interpret Equity_{bt} as another source of funding for the bank that is different than deposits. It could well be thought of as a generic source of funding.

3. Banks problem. Taking $R_k^d(\mathbb{S})$, banks set $R_k(\mathbb{S})$ to maximize profits.

4. Market Clearing. $w(\mathbb{S})$, $R_k^d(\mathbb{S})$, are such that labor market clears $L^s(\mathbb{S}) = \int l(z, a, \mathbf{T}, \mathbb{S}) X(dz, da, d\mathbf{T})$, and banks' balance sheet holds $Deposits_k(\mathbb{S}) + Equity_k = Loans_k(\mathbb{S})$

4.2 Solution Method

Since there are only a handful of banks, a shock to the financial conditions of a bank will create aggregate disturbances. Therefore, when agents are formulating their policy functions, they need to forecast the behavior of the input prices in the economy—namely, the wage rate and the deposit rate at each bank. In order to do so, agents need to forecast the behavior of the cross-sectional distribution of entrepreneurs and banks, which is an infinite-dimensional object. I take advantage of methods developed by Ahn, Kaplan, Moll, Winberry, and Wolf (2018). In particular, the solution will be globally accurate with respect to the individual state space, and will be a linear approximation with respect to the aggregate shocks.

Agents in the model do not expect any bank funding shocks, and after the realization of the shock there is perfect foresight. An idiosyncratic bank liquidity shock, which I model as a reduction in the equity value of the bank has aggregate consequences, so it is treated as an aggregate shock. The dimension-reduction techniques of Ahn et al. (2018) are key to solve the model efficiently.

5 Estimation

The parametrization of the model takes two steps. The majority of the parameters are calibrated. Most of these parameters are well studied and I fix them at standard values. I use microdata to calibrate a subset of parameters that are not widely used in macroeconomic models but for which we have good evidence. Then, the key parameters of the model, θ

and φ, are estimated to target the patterns observed in cross-sectional studies of the bank lending channel that were introduced in the previous sections.

I offer a preview of the results of this section. In the benchmark calibration, the values of θ and φ I estimate are low, implying low ability to adjust to bank shocks. As an illustration, Under an alternative specification of the labor market, (high α), the values of θ and φ that are consistent with the cross-sectional elasticities are large.

On top of evidence from labor economics that advocates for an economy with a low α, I use an additional cross-sectional moment from the banking literature as a sanity check. I extend the model to have two symmetric regions. In models with flexible labor markets within the region (α is high), the indirect effects of bank shocks are positive. This means that a firm without exposure to a shocked bank in a region where the average exposure to the troubled bank is high will outperform an unexposed firm in a region where the average exposure to the troubled bank is low. This prediction is at odds with the evidence, as Huber (2018) has documented. Only when there are substantial rigidities in local labor markets, the model is consistent with the sign of the indirect effect. Therefore, the model rejects the limit of high α, consistent with the micro evidence from labor economics.

5.1 Calibration of Standard Parameters

Table 1 lists the parameters that I fix throughout the estimation. The intertemporal elasticity of substitution is set to a standard value of 1/2. The Frisch elasticity of labor supply is 0.75, as suggested by Chetty et al. (2011). This value is significantly lower than what is used in most macro models. A highly elastic labor supply will increase the aggregate effects of a bank shock, by making it more difficult for wages to go down after a negative shock, increasing the elasticity of output to bank funding shocks.

I set η, the elasticity of substitution across goods equal to 4, within the range of estimates in Broda and Weinstein (2006). I set the discount rate ρ equal to 0.03 per year as in Itskhoki and Moll (2019). I set the persistence of the shock ρ_E at 0.95, consistent with

Parameter	Description	Value
$1/\gamma$	Intertemporal Elasticity of Substitution	1/2
ρ	Discount Rate	0.03
η	Elasticity of Substitution - Goods Market	4
$1/\phi$	Frisch Elasticity of Labor Supply	0.75
z	Two-State Markov Process	0.9 - 1.1
λ	Intensity of Poisson productivity shock	1/3
$\mathcal{B}$	Number of Banks in the Economy	10
ρ_E	Persistence of Equity Shock	0.95
d	Distance Coefficient	3 bank relationships
χ	Elasticity of deposits to deposit rates	5

Table 1: Externally calibrated parameters

Note: The table presents the parameters of the model that I calibrate externally.

the persistence used by Gertler and Kiyotaki (2015). I set the parameters of the productivity Poisson process to target the volatility of 0.056 and a persistence of 0.9 as chosen by Winberry (2018).

I set the number of banks in the economy $N_{\mathcal{B}}$ equal to 10 equal-sized banks. This number replicates the across-Metropolitan Statistical Area median Herfindahl-Hirschmann Index (HHI) of 0.11 coming from data from the Community Reinvestment Act (CRA) data that report business loans for 2006 in the U.S. As an alternative, using call reports data at the national level, the HHI of commercial and industrial loans (C&I) for 2006 is 0.05, implying 20 equal-sized banks. However, this number underestimates the degree of concentration in C&L loans, since firms prefer banks that are closer to them (see Nguyen (2019)). The parameter d, controls how many banking relationships each firm will have. I fix d so that firms have three banking relationships on average, as reported by Huber (2018). I set χ, the parameter that governs how much deposits flow out of a bank with lower deposit rates to 5, matching the semi-elasticity reported by Drechsler et al. (2017).

Moment	Source	Value
Bank Credit Elasticity	Huber (2018)	-0.166
Output Elasticity	Huber (2018)	-0.044

Table 2: Microeconomic Targets

Note: This table shows the two statistics I will target with θ and φ. These are the values of cross-sectional elasticities on firm employment and credit introduced in the identification section. The targets come from Huber (2018).

5.2 Estimation of Key Parameters

Using the relative effects in the data as target moments to estimate the full model, I structurally estimate the parameters values for θ, the elasticity of substitution of firms across banks, and φ, the elasticity at which firms switch away from bank credit. The idea behind the identification is the same as exposed in the identification section, with the difference that the full model gives dynamics to simulate a simulated panel dataset, and that the model is globally accurate with respect to individual policy functions, which are more accurate than the second-order Taylor expansions we introduced before. Specifically, I simulate a panel of firms over time after a bank funding shock. With the simulated data, I run a regression analysis that replicates the cross-sectional analysis, after collapsing a set of periods before and after the shock into two bins, the pre-period and the post-period. Table (2) specifies the microeconomic targets of the calibration. For a detailed discussion of the regressions, please refer to the identification section.

5.3 Estimation of the Equity Shock

As shown in the previous sections, the size of the cross-sectional estimates depend on the size of the shock. Therefore, to estimate the model targeting these cross-sectional patterns it is important to feed an equity shock of the right size. I replicate the drop of the book value of equity of Commerzbank reported by Huber (2018) since I am using the cross-sectional estimates in that paper to parameterize the model. In particular, I feed a decline of equity in one bank as large as Commerzbank of 68% of its value, consistent with the equity losses experienced by this bank between 2007 and 2009.

5.4 Sensitiviy of Cross-Sectional Elasticities to Structural Parameters

Before showing the estimation of the model, I illustrate the effect of θ and φ in determining the cross-sectional moments and the effect of different values of α in shifting the effect of these two parameters.

Figures 4 and 5 show the effect of changing θ for two values of α, on the cross-sectional moments of credit and production, respectively, while keeping the rest of the parameters in the model fixed. As is intuitive from previous sections, a higher value of θ, by increasing the flexibility of firms on switching across banks, decreases the cross-sectional elasticities of both output and credit. In the limit, where $\theta \to \infty$, the elasticities tend to zero. Figures 4 and 5 make an additional point. Because the elasticity is larger in absolute value when labor markets do not have any frictions, the value of θ that is consistent with a given elasticity is significantly larger when $\alpha \to \infty$ than when α is low. Therefore, in order to match the same cross-sectional elasticities, θ will be lower in an economy with labor and demand insubstitutabilities.

Figures 6 and 7 perform the same exercise for the elasticity at which firms move away from bank credit (φ). These figures show that the identification argument holds beyond the second order approximation we did in the simple model. When φ increases the output effects of the shock are smaller, but the credit effects of the same shock are larger.

With respect to α, Figure (7) shows that for frictionless labor markets, the value of φ that is consistent with a given elasticity is higher than for markets with frictions. The intuition for this result is the same as for the results that involved θ. Under a frictionless labor market, the cross-sectional effects are larger since it is easier to move labor across firms. In the case of Figure (6), when α is larger, which increases the losses of a given shock, firms move away from credit by more, explaining why the schedule of $\alpha = 1000$ is below from the schedule for $\alpha = 1$.

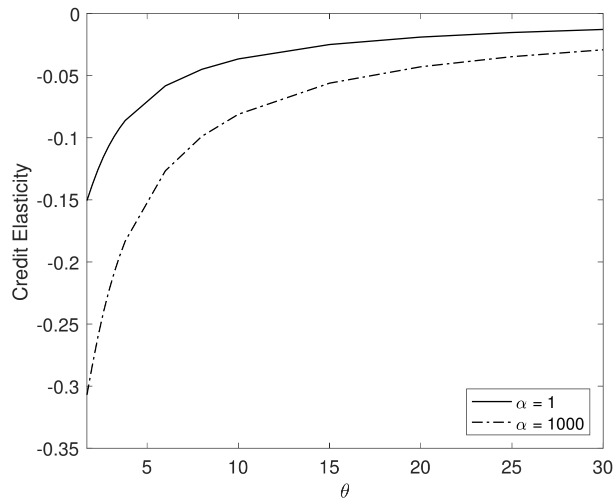

Figure 4: Sensitivity of the cross-sectional elasticity of credit on θ.

Note: This figure shows the cross-sectional elasticity of credit in response to a bank shock for different values of θ, the elasticity of substitution of funding across banks. I conduct this exercise for two different values of α: first for a market with $\alpha \to \infty$, and second, for a low level of α when there are substantial difficulties in moving labor across firms.

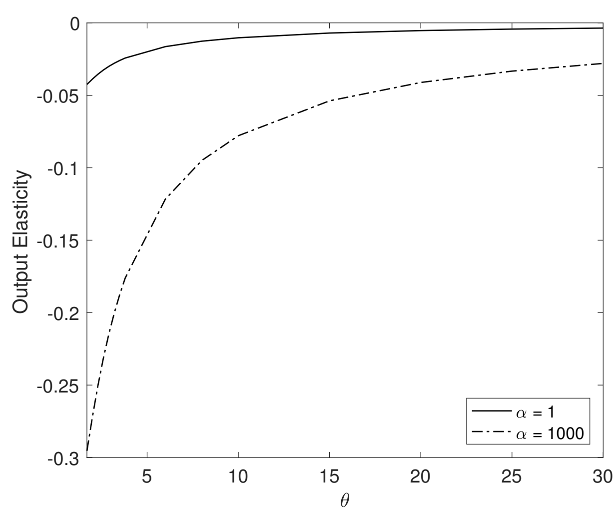

Figure 5: Sensitivity of the cross-sectional elasticity of output on θ.

Note: This figure shows the cross-sectional elasticity of output in response to a bank shock for different values of θ, the elasticity of substitution of funding across banks. I conduct this exercise for two different values of α: first for a market with $\alpha \to \infty$, and second, for a low level of α when there are substantial difficulties in moving labor across firms.

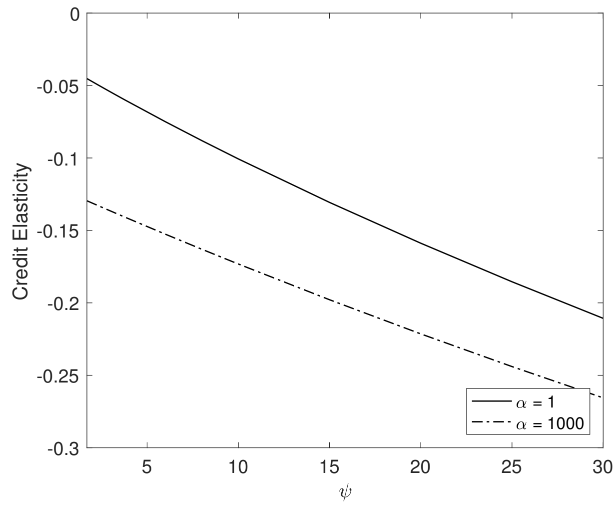

Figure 6: Sensitivity of the cross-sectional elasticity of credit on φ

Note: This figure shows the cross-sectional elasticity of credit in response to a bank shock for different values of φ, the elasticity of substitution of bank credit. I conduct this exercise for two different values of α: first for a market with $\alpha \to \infty$, and second, for a low level of α when there are substantial difficulties in moving labor across firms.

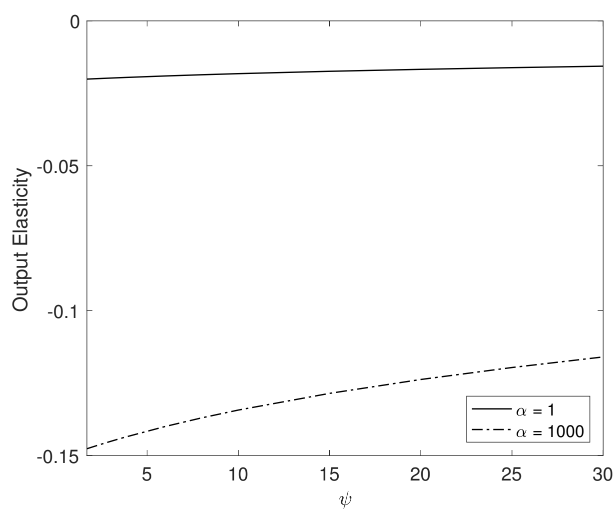

Figure 7: Sensitivity of the cross-sectional elasticity of output on φ

Note: This figure shows the cross-sectional elasticity of output in response to a bank shock for different values of φ, the elasticity of substitution of bank credit. I conduct this exercise for two different values of α: first for a market with $\alpha \to \infty$, and second, for a low level of α when there are substantial difficulties in moving labor across firms.

Parameter	Description	Value ($\alpha = 1$)	Value ($\alpha = 1000$)
θ	Substituability Across Banks	1.5	6.5
φ	Inverse credit Dependence	4.5	20

Table 3: Estimated Elasticities of Substitution

Note: This Table shows the values of θ and φ that target the cross-sectional elasticities shown in Table 2. I estimate θ and φ for two values of α. A benchmark case when $\alpha = 1$ and another when $\alpha \to \infty$, which reflects an economy where there is perfect mobility of labor across firms.

6 Estimated Parameters

In this section I report the combination of θ and φ that match the values of the observed moments as reported in Table 2. I report the values that fit the cross-sectional moments in models where $\alpha = 1$ and $\alpha \to \infty$, with the purpose of showing that the estimated structural parameters are vastly different depending on the assumed structure of the labor market.

The estimated parameters in Table (3) led me to reject that firms and banks operate in markets of perfect substituability, which is the limit of $\theta \to \infty$ and $\varphi \to \infty$. The numbers in the table alone do not tell us quantitatively, how important are deviations from perfect substituability, an answer that I provide in the next section.

Table (3) makes clear the importance of the structure of the labor market. Under elastic labor markets, the parameters are larger, implying that firms are more flexible in reacting to a bank shock. Therefore the effects of bank shocks will be lower.

We have shown how α, the parameter that governs the extent of frictions in the labor market, is important in this model. The reason is that the extent of real rigidities in the model change the extent to which demand and inputs can be reallocated across firms. When there are substantial frictions in reallocating labor across firms, the model requires substantial frictions in banking as well, in order to match the cross-sectional moments. On the other side, with frictionless labor markets, the banking sector must be relatively flexible, or the model would predict cross-sectional elasticities that are larger than the ones observed in the data. The question becomes how to distinguish across values of α.

I use two sources of evidence: direct evidence on the value of α, and indirect evidence showing that additional cross-sectional patterns in the banking sector reject the case of labor markets with low frictions.

In particular, Webber (2015) document an inelastic firm-specific labor supply. This evidence has already been used in the literature by Chodorow-Reich (2014), and I show that in a more flexible model with flexible patterns of substitution of firm funding, the extent of these frictions is still important. I also use an additional cross-sectional moment, the *indirect effects* of bank lending cuts, to distinguish across models. The indirect effects measure how a firm without direct exposure to the shocked bank that operates in a region where other firms are highly exposed behaves with respect to another firm without direct exposure to the troubled bank that operates in a region where firms are not highly exposed to the troubled bank. Huber (2018) reports that the indirect effects of bank-lending cuts are negative. This means that unexposed firms in exposed regions underperform unexposed firms in unexposed regions.

I extend the model to illustrate the behavior of the indirect effects. Specifically, I extend the model to have 2 symmetric regions. The regions are segmented in the markets for goods and labor. That is, each firm produces non-tradeable goods, and people cannot move across regions. However, there is partial financial integration. Lending relationships are determined by distance, regardless of geographical barriers. Therefore, firms may borrow from banks in their home or a foreign region, but must sell their products and hire their workers in the local region. As before, the extent to which workers can move across firms *within the same region* is given by the parameter α:

$$\Delta \log Y_{jr} = \beta_0 + \beta_1 \nu_{jr,pre} + \beta_2 \nu_{jr,pre}^{-} + \epsilon_{jr}. \tag{33}$$

Equation (33) presents the regression we will run to get the reduced-form indirect effects. The dependent variable is the log change of an outcome of interest (in this case output) of firm j located in region r, and the right-hand-side variables are the pre-existing lending relationship of the same firm and the average exposure of the firms in region r. β_2 is the coefficient of interest; it captures the change in outcomes of a firm with $\nu_{jr,pre} = 0$ in a region where the average exposure is complete $\nu_{j\bar{r},pre} = 1$, with respect to a firm with zero direct exposure $\nu_{j-r,pre} = 0$ in a region $-r$ where the average exposure is also zero $\nu_{j\bar{r},pre} = 0$.

To give a clear sense of the effect of α in the model, I show the effect of different values of this parameter on the three cross-sectional patterns I have documented so far: the elasticity of credit, the elasticity of output, and the indirect effects. In order to provide a clean intuition, I fix all the other values of the parameters at arbitrary values, including θ and φ. This approach is in contrast to the previous results where I estimated φ and θ for different values of α.

Figures (8) and (9) illustrate an argument that is familiar by now. When labor markets exhibit less frictions, the direct cross-sectional effects increase in absolute value. This happens because the wedge between marginal costs between firms with and without exposure to the shock increases. As a consequence, the wedge between prices, production, and credit demand increases as well.

Figure (10) plots the indirect effects of the lending shock for different values of α. The figure makes clear that as labor markets become more efficient, the indirect effects of a lending shock become more positive. That is, an unexposed firm in an exposed region experiences a outperforms an unexposed firm in an exposed region. On the contrary, Huber (2018) reports that firms in exposed regions underperform unexposed firms in exposed regions. Although the confidence intervals on the indirect effects reported by Huber (2018) are wide, they reject positive values of the indirect effects, which means that the model rejects values of α greater than 1.

The insight that the model rejects perfectly competitive labor markets by using the indirect effects is key in the estimation of the aggregate effects of bank shocks. As Figure (10) shows, only values of $\alpha < 1$ can rationalize negative indirect effects. Therefore, we

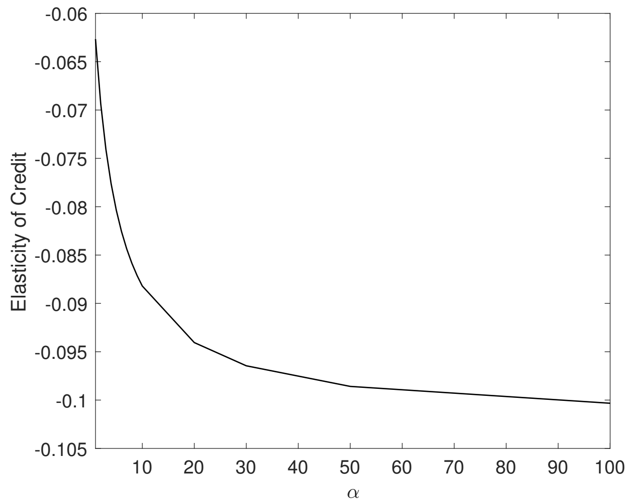

Figure 8: Sensitivity of the cross-sectional effects on credit of an idiosyncratic bank shock to α

Note: This Figure shows the cross-sectional effect on credit to a bank shock for different values of α, the extent of frictions in the labor market. All the other parameters are fixed in their calibrated values, except θ and φ which are fixed in an arbitrary level of 5. The qualitative properties of the figure do not depend on this choice.

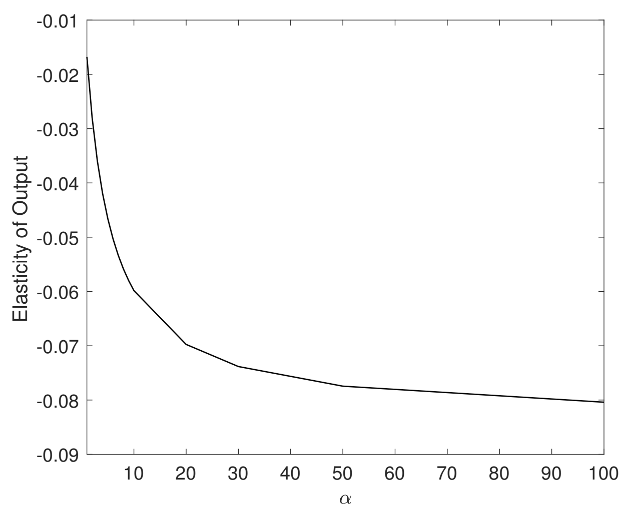

Figure 9: Sensitivity of the cross-sectional effects on output of an idiosyncratic bank shock to α

Note: This Figure shows the cross-sectional effect on ouput to a bank shock for different values of α, the extent of frictions in the labor market. All the other parameters are fixed at their calibrated values, except θ and φ which are fixed in an arbitrary level of 5. The qualitative properties of the figure do not depend on this choice.

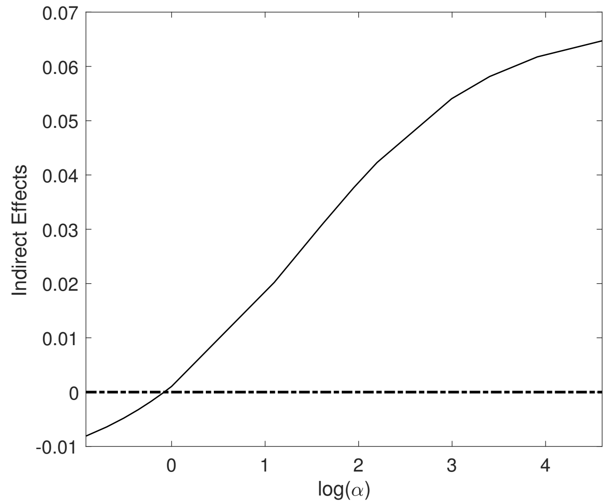

Figure 10: Sensitivity of the indirect effects on credit of an idiosyncratic bank shock to α.

Note: This figure shows the indirect effects of a bank shock for different values of α, the extent of frictions in the labor market. All the other parameters are fixed at their calibrated values, except θ and φ , which are fixed at an arbitrary level of 5. The qualitative properties of the figure do not depend on this choice.

can reject the limit of frictionless labor markets, and with it, the small elasticities of output to lending they entail.

7 Discussion

7.1 The Aggregate Effects of Bank Supply Shocks

In this section I analyze the aggregate effects of a cut in the supply of bank lending. In particular I compute the ratio between the integral of the discounted value of aggregate output drops over the integral of the discounted value of the funding shock. Formally, I compute an elasticity ε^M as follows:

$$\varepsilon^M = \frac{\int_0^T e^{-\rho t}\left(\log(Y_t) - \log(\bar{Y})\right) dt}{\int_0^T e^{-\rho t} \log(\text{Lending}_t) - \log(\bar{\text{Lending}}) dt}). \tag{34}$$

The reason to compute the elasticity of output to lending in this way is that output may exhibit different persistence than total lending, and that the shock that is feeding the economy is persistent, inducing additional responses in output and lending beyond the response on impact. Note as well that the elasticity is computed with respect to lending, not with respect to the shock. There are two reasons for this. First, the policy-relevant variable is the reduced ability of banks to make loans—or to put it another way, the drop in the right-hand-side of the balance sheet of the banking sector. Second, this definition admits comparisons with back-of-the-envelope aggregations that cross-sectional studies make by abstracting from general equilibrium effects.

ε^M should be interpreted as the elasticity of output to lending caused by a shock in the supply of bank lending. It is the macroeconomic equivalent of an instrumental variables (IV) specification. In an IV, we compute regressions between two endogenous variables, and find an instrument that affects the right-hand-side variable (lending in this case), and that only affects the dependent variable (aggregate output), through its effect on lending.[2]

The result of this section is an estimation of this elasticity, and I will show the sensitivity of the elasticity for both experiments with respect to the key parameters of the model. As before, we will consider results for two extreme values of α, the extent of rigidities in the labor market.

7.2 The Aggregate Effects of a Symmetric Bank Shock

We start by performing an experiment in which every bank in the economy is shocked at the same time. This experiment is interesting for several reasons. One, this type of shock captures the attention of macroeconomists and policy experts. Second, it speaks to situations without meaningful heterogeneous exposure to the shock, where using the cross-section to estimate effects is implausible. However, we will inquire how the knowledge of the structural parameters we gained from the cross-sectional estimates extrapolates to an aggregate shock.

Figures (11) and (12) show the effects of the key parameters, φ and θ, in determining

[2] Computing an elasticity between two endogenous variables in macroeconomics is commonplace. The Phillips Curve slope for instance is the elasticity of inflation to unemployment caused by a demand shock. Interest rate parities relates exchange rates to interest rate differentials.

Calibration	$\alpha = 1$	$\alpha = 1000$
Benchmark (%)	19.63	6.73

Table 4: Elasticity of Aggregate Output to Aggregate Bank Lending

Note: This table shows the elasticity of output to lending to bank lending. Each column shows the elasticity of output to bank lending for two assumptions of the labor market. One where there are meaningful frictions in the labor market ($\alpha = 1$), and for a case where labor markets are frictionless.

the output effects of an idiosyncratic shock. The x-axis of these figures is the value of one parameter, and the y-axis is the elasticity of aggregate output to aggregate lending after an aggregate bank shock. The solid line shows the preferred case when $\alpha = 1$, and the dashed line shows the case of frictionless labor markets, when $\alpha \to \infty$. The marker in each line shows the estimated value of the parameter for each case.

Figure (11) shows that higher values of φ, which decrease the extent of financial frictions, diminishes the elasticity of output to lending. Under frictionless labor markets, the estimated parameter of 20, implies that the elasticity of output to lending is one third the elasticity estimated when there are meaningful frictions in the labor market. The solid and dashed line are over the other for two indicating that other than φ, no other parameters that differ across the two parametrizations of the model α or θ change the size of the elasticity.

On the other side, Figure (12) shows that θ is not quantitatively relevant for determining the aggregate elasticity since the lines are flat around the estimated values. This is true even when θ is relevant at determining the cross-sectional responses, as shown in previous sections. This result indicates that irrespective of the value of θ, the response of output to lending is the same. It does not mean that θ is irrelevant in the aggregate. To think about this issue it is useful to remember that the elasticity of output to lending is equal to the elasticity of output to the shock, divided by the elasticity of lending to the shock. The flatness of the elasticity of output to lending indicates that the behavior of lending follows the same pattern.

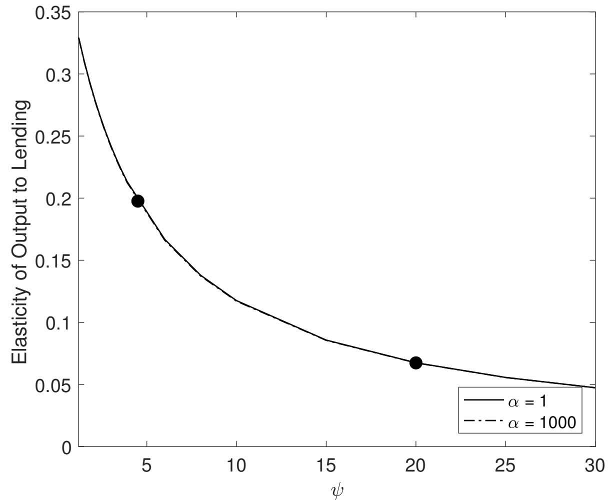

Figure 11: Sensitivity of the aggregate effects of an aggregate bank shock to φ

Note: This Figure shows the aggregate output drop after an idiosyncratic bank shock for different values of φ, the elasticity of credit dependence. We perform this exercise for two different values of α. First for a frictionless labor market, where $\alpha \to \infty$. And second, for a low level of α when there are substantial frictions in the labor market. All the parameters are fixed in their calibrated or estimated values except for φ. The dot on each line represents the estimated value for φ and the correspondent output drop.

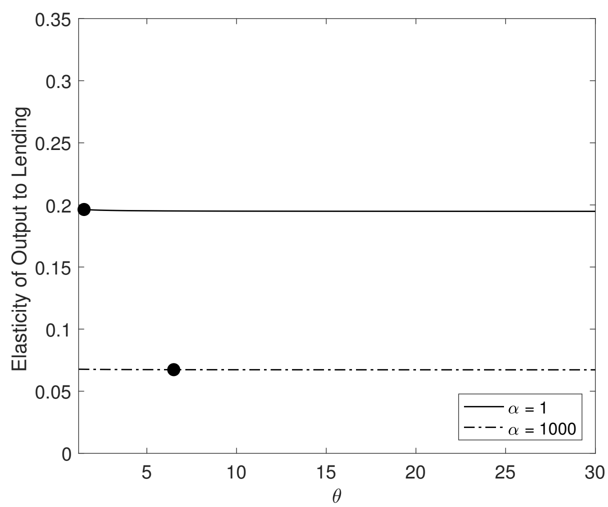

Figure 12: Sensitivity of the aggregate effects of an aggregate bank shock to θ

Note: This Figure shows the aggregate output drop to a bank shock for different values of θ, the substituability of funds across banks. We perform this exercise for two different values of α. First for a frictionless labor market, where $\alpha \to \infty$. And second, for a low level of α when there are substantial frictions in the labor market. All the parameters are fixed in their calibrated or estimated values except for θ. The dot on each line represents the estimated value for φ and the correspondent output drop.

7.3 The Aggregate Effects of a Granular Bank Shock

So far, I presented results about the effects on aggregate output of a cut in the supply of bank lending of the whole banking sector, a truly aggregate shocks. However, granular bank lending cuts have aggregate consequences in the model. The reason is that banks in the model are large entities. In this section I illustrate the macroeconomic effects of an idiosyncratic bank shock. I measure the elasticity of aggregate output to the cut in the supply of bank lending of one entity with the following elasticity:

$$\varepsilon^{M,b} = \frac{\int_0^T e^{-\rho t} \left(\log(Y_t) - \log(\bar{Y})\right) dt}{\int_0^T e^{-\rho t} \log(\text{Lending}_{bt}) - \log(\bar{\text{Lending}}_b) dt}). \tag{35}$$

Where $\varepsilon^{M,b}$ is the macro elasticity of output after a cut in lending of bank b. The interpretation of the elasticity is the same as before. It is the macroeconomic equivalent of an instrumental variable regression, where after taking a stance in a source of variation, we compare the effect of that shock on two exogenous variables.

The main result of this section is that opposed to the case of a truly aggregate shock, in this case, θ the elasticity of substitution of funds across different banks is important in determining aggregate outcomes. The economic intuition behind this result is clear. When one bank suffers a given shock that induces the bank to offer less attractive loan terms to its customers, the elasticity at which firms switch away from the affected bank dictates their change in marginal costs and their output as a consequence. This result is the numerical equivalent of the qualitative argument presented in the theoretical sections of the paper, that shows that when one bank is disrupted, both θ and φ are important in determining the aggregate response of output.

Figure (13) shows on the x axis the elasticity of substitution away from bank credit, and on the y axis, the elasticity of aggregate output to idiosyncratic bank lending. Here, I estimate an elasticity of 0.025, which means that if the shocked bank (that had a bank share of 10 percent) cuts its lending by 1 percent, then aggregate output will fall by 0.025

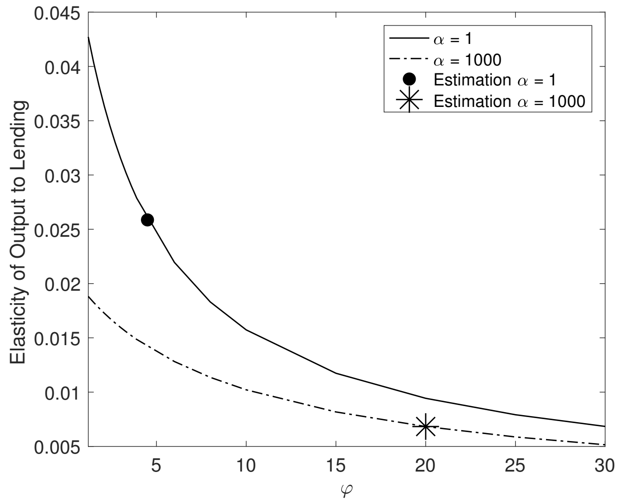

Figure 13: Sensitivity of the aggregate effects of an idiosyncratic bank shock to φ

Note: This Figure shows the aggregate output drop after an idiosyncratic bank shock for different values of φ, the elasticity of credit dependence. We perform this exercise for two different values of α. First for a frictionless labor market, where $\alpha \to \infty$. And second, for a low level of α when there are substantial frictions in the labor market. All the parameters are fixed in their calibrated or estimated values except for φ. The dot on each line represents the estimated value for φ and the correspondent output drop.

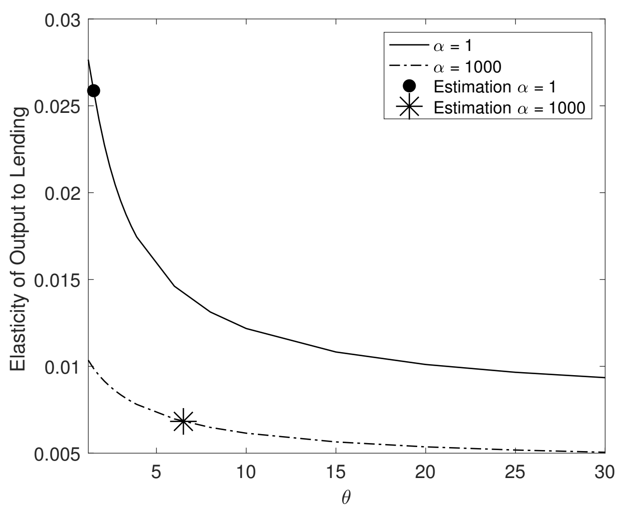

Figure 14: Sensitivity of the aggregate effects of an idiosyncratic bank shock to θ

Note: This Figure shows the aggregate output drop to a bank shock for different values of θ, the substituability of funds across banks. We perform this exercise for two different values of α. First for a frictionless labor market, where $\alpha \to \infty$. And second, for a low level of α when there are substantial frictions in the labor market. All the parameters are fixed in their calibrated or estimated values except for θ. The dot on each line represents the estimated value for φ and the correspondent output drop.

percent. The figure also shows that when $\alpha \to \infty$, the case of perfect labor mobility, this elasticity would be roughly 0.007.

Figure (14) shows on the x axis the elasticity of substitution across banks, and on the y axis, the elasticity of aggregate output to idiosyncratic bank lending. This figure makes clear that θ, the elasticity of substitution across banks, is important in determining the aggregate response of aggregate output to an idiosyncratic bank shock.

The fact that the elasticity is lower is no surprise, as illustrated in the theoretical section of the paper, the effect of a disruption of one bank is weighted by its market share in the pre-period. What is worth emphasizing is that the elasticity of substitutition of funding across banks is now relevant to determine aggregate fluctuations. The estimation of the model suggests that a 10 percent drop in lending of a bank with 10 percent market share would generate a drop in aggregate activity of 0.25 percent.

7.4 Comparing General to Partial Equilibrium

An important use of the parametrized model is to compare the estimated aggregate bank-lending channel to the alternative measure when general equilibrium effects are ignored. These aggregations are important because after estimating a result in the cross-section using micro data and regression analysis, empirical researchers want to assess the potential of their findings to have aggregate implications.

Back-of-the-envelope (partial equlibrium) aggregations measure the difference in any given firm outcome between each firm in the economy with respect to the least exposed firm to the shock, a control firm which we denote with c. In the model we can present an intertemporal version of the partial equilibrium aggregation in present value given by the following expression

$$\varepsilon^{cs} = \frac{\int_0^T e^{-\rho t} \int_0^1 \left(\log(Y_{jt}) - \log(Y_{ct})\right) dj dt}{\int_0^T e^{-\rho t} \int_0^1 \log(\text{Borrowing}_{jt}) - \log(\text{Borrowing}_{ct}) dj dt}. \tag{36}$$

To compare the general and partial equilibrium aggregations, I simulate an experiment in which I shock only one bank. The parametrization of the model indicates that the partial equilibrium aggregation (ε^{cs}) is 10 percent higher than the general equilibrium response (ε^{M}). This message is important. The preferred estimation of the model, that is consistent with many patterns documented over the years in the corporate finance literature, indicates that general equilibrium forces of the model do *not* cause the micro patterns to vanish in the aggregate.

However this result does not need to hold, and it depends on the parameters we have estimated. For instance, under an alternative model with frictionless labor market frictions, the partial equilibrium aggregation is only one fifth of the general equilibrium effect. Meaning that extrapolating from cross-sectional estimates in such a world would lead researchers to overestimate the relevance of the firm credit channel by a factor of five. However, such a world with frictionless labor markets is at odds with the evidence.

Figure (15) shows how the extent of financial frictions in the model, the substitution from bank credit (φ), and the θ, change the ratio between the general equilibrium and the partial equilibrium elasticities for two parametrizations of the labor market. In particular, it shows that the General Equilibrium aggregation can be higher or lower than the partial equilibrium one as θ and φ change. It also shows that general equilibrium effects are stronger when labor markets work better, as illustrated in the theoretical sections of the paper. It shows that the ratio between general equilibrium and partial equilibrium elasticities is more or less stable, and higher for a model with input market frictions.

However, although Figure (15) presents important information with respect to the output effects of a given lending drop, it does not answer the question of whether back-of-the-envelope aggregations over or underestimate drops in output. The reason is that for the same shock, the aggregate and the cross-sectional drop in *lending* are different. Specifically, Figure (15) shows that for each 1 percent of a lending drop caused by the shock, output reacts by with a given elasticity. However the two aggregations differ in the percent change in lending they exploit. The general equilibrium aggregation exploits the drop in aggregate lending, while the partial equilibrium one exploits the differential change in lending across banks.

Figure (16) shows the ratio of the output aggregations, which means the ratio of the numerators of ε^M and ε^{cs}. The figure makes several points. First, it shows that across the parameter space, in principle the general equilibrium effects on output can be larger, similar, or smaller than is implied by partial equilibrium estimates. However, the estimation of the model imposes restrictions on the size of the difference. In my benchmark estimation the output drops in GE are around 70 percent those implied in PE.. In an alternative calibration with perfect mobility in labor the ratio would be around 1/6. That is, labor market immobility elevates the ratio of the output effects in GE to PE aggregations by a factor of 4.

8 Conclusion

The aggregate effects of cuts in the supply of bank lending are difficult to measure using aggregate time-series because bank funding disruptions coincide with other shocks that affect loan demand, and because banks are sensitive to drops in economic conditions creating reverse causality concerns.

Cross-sectional effects solve many of these reverse causality problems, but face an aggregation problem. Because they compare treated to untreated units, they difference general equilibrium margins that are important for aggregate allocations.

Using direct and indirect evidence on the cost of reallocating inputs across firms, and on the relative effects of bank shocks on firm outcomes and credit, I conclude that the aggregate consequences of bank lending cuts are large. When lending drops by 1 percent due to a disruption in bank funding, aggregate output is reduced by 0.2 percent.

This elasticity depends on the extent of bank dependence, and this paper uses cross-

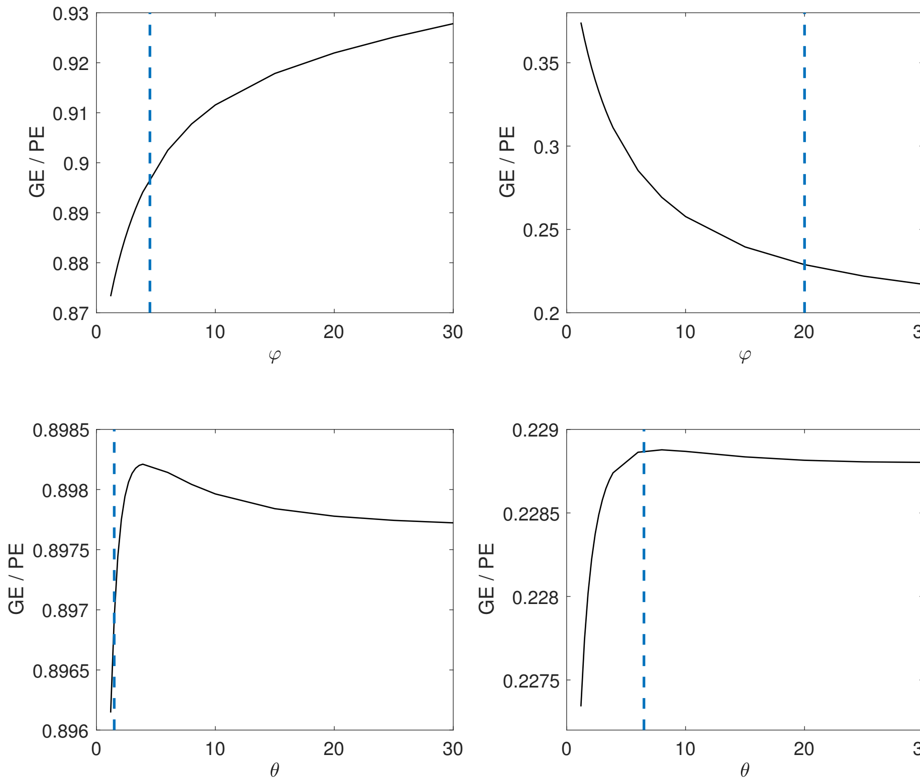

Figure 15: Ratio of the aggregate elasticity to back-of-the-envelope aggregations

Note: This figure shows four panels. The left column shows figures when there are significant frictions in the labor market $\alpha = 1$. The right column shows the case when $\alpha \to \infty$. The top row shows results for the elasticity of substitution away from bank credit φ, while the bottom row shows results for the elasticity at which firms substitute funding from a particular bank, θ. Each panel shows the ratio between the elasticity of aggregate output to aggregate bank lending (ε^M), to the back-of-the-envelope aggregation ε^{cs}. The x axis shows the value of a parameter keeping constant all the other parameters in the parametrization.

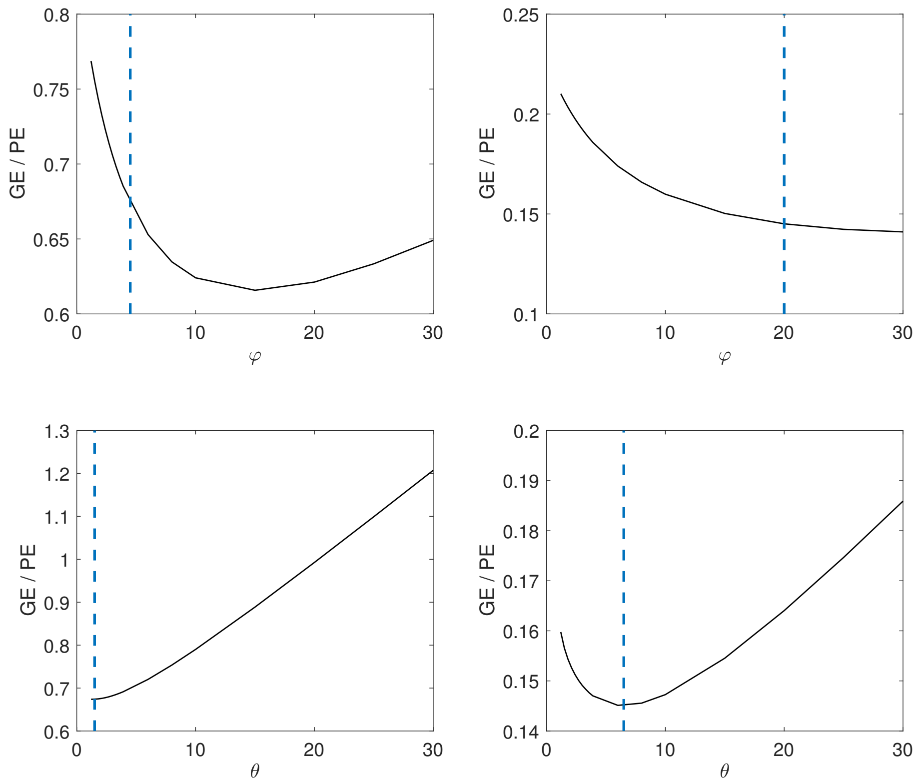

Figure 16: Ratio of the aggregate output drop with respect to back-of-the-envelope aggregations

Note: This figure shows four panels. The left column shows figures when there are significant frictions in the labor market $\alpha = 1$. The right column shows the case when $\alpha \to \infty$. The top row shows results for the elasticity of substitution away from bank credit φ, while the bottom row shows results for the elasticity at which firms substitute funding from a particular bank, θ. Each panel shows the drop of aggregate output to the drop in output inferred from a back-of-the-envelope-aggregation. The x axis shows the value of a parameter keeping constant all the other parameters in the parametrization.

sectional evidence to recover this elasticity. Taking a stance on the frictions needed to reallocate inputs and demand across firms is important, even under the experiment of an aggregate shock where all firms are shocked symmetrically. This happens because in order to target the same cross-sectional moments, elastic input and demand markets require banking frictions to be milder than in an economy with substantive frictions in reallocating inputs and demand.

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Appendix for Online Publication

A Proofs for Online Publication

A1 Derivation of Aggregate Output in the simple model

Firms' j profit maximization can be written as:

$$\pi_j = y_j p_j - w_j R_j l_j$$

s.t.

$$y_j = z_j \left[\int_0^1 l_j^{\frac{\sigma-1}{\sigma}}(w)dw \right]^{\frac{\sigma}{\sigma-1}}$$
$$p_j = \left(\frac{y_j}{Y}\right)^{-1/\eta}$$

where R_j is the average financing cost of the firm.

Replacing the constraints into the profits function and maximizing with respect to l_j, while taking w_j as given, yields:

$$\max_{l_j} \pi_j = y_j^{\frac{\eta-1}{\eta}} Y^{\frac{1}{\eta}} - w_j R_j l_j$$
$$= (z_j l_j)^{\frac{\eta-1}{\eta}} y^{\frac{1}{\eta}} - w_j R_j l_j$$

The first order condition is:

$$\frac{\eta-1}{\eta} z_j^{\frac{\eta-1}{\eta}} Y^{\frac{1}{\eta}} l_j^{-\frac{1}{\eta}} = w_j R_j$$

Re-arranging:

$$\left(\frac{z_j l_j}{y}\right)^{-\frac{1}{\eta}} = w_j R_j \frac{\eta}{\eta-1} z_j^{-1}$$
$$\frac{z_j l_j}{y} = w_j^{-\eta} R_j^{-\eta} (\frac{\eta}{\eta-1})^{-\eta} z_j^{\eta}$$
$$l_j = w_j^{-\eta} R_j^{-\eta} (\frac{\eta}{\eta-1})^{-\eta} z_j^{\eta-1} y$$

Optimal firm-level labor demand is given then by,

$$l_j = w_j^{-\eta} \;\; R_j^{-\eta} \;\; \left(\frac{\eta}{\eta-1}\right)^{-\eta} \;\; z_j^{\eta-1} \;\; Y \tag{37}$$

Now, plugging in the labor supply equation, $w_j = w\left(\frac{l_j}{L}\right)^{\frac{1}{\alpha}}$, which firms take as given yields

$$\begin{aligned} l_j = & \;\; R_j^{-\eta} \;\; \left(\frac{\eta}{\eta-1}\right)^{-\eta} \;\; z_j^{\eta-1} \;\; Y \;\; w^{-\eta} l_j^{-\frac{\eta}{\alpha}} L^{\frac{\eta}{\alpha}} \\ = & \left(\frac{\eta}{\eta-1}\right)^{-\frac{\eta\alpha}{\alpha+\eta}} \quad Y^{\frac{\alpha}{\alpha+\eta}} \;\; z_j^{(\eta-1)\frac{\alpha}{\alpha+\eta}} R_j^{-\eta\frac{\alpha}{\alpha+\eta}} \;\; w^{-\eta\frac{\alpha}{\alpha+\eta}} \;\; L^{\frac{\eta}{\alpha+\eta}} \end{aligned}$$

By elevating to the power $\frac{\alpha+1}{\alpha}$, integrating over firms, and elevating to the power $\frac{\alpha}{\alpha+1}$, we get an expression for aggregate labor, $L = \left[\int_0^1 l_j^{\frac{\alpha+1}{\alpha}} dj\right]^{\frac{\alpha}{\alpha+1}}$.

$$\begin{aligned} L &= \left(\frac{\eta}{\eta-1}\right)^{-\frac{\eta\alpha}{\alpha+\eta}} Y^{\frac{\alpha}{\alpha+\eta}} w^{-\eta\frac{\alpha}{\alpha+\eta}} L^{\frac{\eta}{\alpha+\eta}} \left[\int_0^1 [z_j^{(\eta-1)\frac{\alpha}{\alpha+\eta}} R_j^{-\eta\frac{\alpha}{\alpha+\eta}}]^{\frac{\alpha+1}{\alpha}} dj\right]^{\frac{\alpha}{\alpha+1}} \\ L^{\frac{\alpha}{\alpha+\eta}} &= \left(\frac{\eta}{\eta-1}\right)^{-\frac{\eta\alpha}{\alpha+\eta}} Y^{\frac{\alpha}{\alpha+\eta}} w^{-\eta\frac{\alpha}{\alpha+\eta}} \mathbb{E}[z_j^{(\eta-1)\frac{\alpha+1}{\alpha+\eta}}]^{\frac{\alpha}{\alpha+1}} \mathbb{E}[R_j^{-\eta\frac{\alpha+1}{\alpha+\eta}}]^{\frac{\alpha}{\alpha+1}} \end{aligned}$$

Lastly,

$$\begin{aligned} L &= \left(\frac{\eta}{\eta-1}\right)^{-\eta} Y \;\; w^{-\eta} \;\; \mathbb{E}[z_j^{(\eta-1)\frac{\alpha+1}{\alpha+\eta}}]^{\frac{\alpha+\eta}{\alpha+1}} \;\; \mathbb{E}[R_j^{-\eta\frac{\alpha+1}{\alpha+\eta}}]^{\frac{\alpha+\eta}{\alpha+1}} \\ w^{\frac{1+\eta\phi}{\phi}} &= \left(\frac{\eta}{\eta-1}\right)^{-\eta} Y \;\; \mathbb{E}[z_j^{(\eta-1)\frac{\alpha+1}{\alpha+\eta}}]^{\frac{\alpha+\eta}{\alpha+1}} \;\; \mathbb{E}[R_j^{-\eta\frac{\alpha+1}{\alpha+\eta}}]^{\frac{\alpha+\eta}{\alpha+1}} \end{aligned} \tag{38}$$

Re-arranging

$$w = \left(\frac{\eta}{\eta-1}\right)^{-\frac{\eta\phi}{1+\eta\phi}} Y^{\frac{\phi}{1+\eta\phi}} \;\; \mathbb{E}[z_j^{(\eta-1)\frac{\alpha+1}{\alpha+\eta}}]^{\frac{\alpha+\eta}{\alpha+1}\frac{\phi}{1+\eta\phi}} \;\; \mathbb{E}[R_j^{-\eta\frac{\alpha+1}{\alpha+\eta}}]^{\frac{\alpha+\eta}{\alpha+1}\frac{\phi}{1+\eta\phi}} \tag{39}$$

Using the fact that $y_j = z_j l_j$, we have:

$$y_j = \left(\frac{\eta}{\eta-1}\right)^{-\frac{\eta\alpha}{\alpha+\eta}} \quad Y^{\frac{\alpha}{\alpha+\eta}} \;\; z_j^{\eta\frac{\alpha+1}{\alpha+\eta}} R_j^{-\eta\frac{\alpha}{\alpha+\eta}} \;\; w^{-\eta\frac{\alpha}{\alpha+\eta}} \;\; L^{\frac{\eta}{\alpha+\eta}}$$

Taking the $\frac{\eta-1}{\eta}$ power, integrating over all the firms, and taking the power $\frac{\eta}{\eta-1}$, we get the following expression:

$$Y = \left(\frac{\eta}{\eta-1}\right)^{-\frac{\eta\alpha}{\alpha+\eta}} \quad Y^{\frac{\alpha}{\alpha+\eta}} \;\; w^{-\eta\frac{\alpha}{\alpha+\eta}} \;\; L^{\frac{\eta}{\alpha+\eta}} \;\; \mathbb{E}[z_j^{\frac{\alpha+1}{\alpha+\eta}(\eta-1)}]^{\frac{\eta}{\eta-1}} \mathbb{E}[R_j^{-\frac{\alpha(\eta-1)}{\alpha+\eta}}]^{\frac{\eta}{\eta-1}}$$

Re-arranging we get an expression for output which depends on aggregate labor and wages:

$$Y = \left(\frac{\eta}{\eta-1}\right)^{-\eta} \;\; L \;\; w^{-\alpha} \;\; \mathbb{E}[z_j^{\frac{\alpha+1}{\alpha+\eta}(\eta-1)}]^{\frac{\alpha+\eta}{\eta-1}} \mathbb{E}[R_j^{-\frac{\alpha(\eta-1)}{\alpha+\eta}}]^{\frac{\alpha+\eta}{\eta-1}}$$

Next, we can replace L and w with the expressions we got above. First, replacing L yields:

$$w = \left(\frac{\eta}{\eta-1}\right)^{-1} \quad \mathbb{E}[z_j^{\frac{\alpha+1}{\alpha+\eta}(\eta-1)}]^{\frac{(\alpha+\eta)}{(\eta-1)(\alpha+1)}} \quad \mathbb{E}[R_j^{-\frac{\alpha(\eta-1)}{\alpha+\eta}}]^{\frac{1}{\eta-1}} \quad \mathbb{E}[R_j^{-\frac{\eta(\alpha+1)}{\alpha+\eta}}]^{\frac{1}{\alpha+1}}$$

Plugging in the expression for w from 39 and re-arranging we get:

$$Y = \left(\frac{\eta}{\eta-1}\right)^{-\frac{1}{\phi}} \quad \mathbb{E}[z_j^{\frac{\alpha+1}{\alpha+\eta}(\eta-1)}]^{\frac{(1+\phi)(\alpha+\eta)}{\phi(\eta-1)(\alpha+1)}} \quad \mathbb{E}[R_j^{-\frac{\alpha(\eta-1)}{\alpha+\eta}}]^{\frac{(1+\eta\phi)}{\phi(\eta-1)}} \quad \mathbb{E}[R_j^{-\frac{\eta(\alpha+1)}{\alpha+\eta}}]^{\frac{(1-\phi\alpha)}{\phi(\alpha+1)}} \tag{40}$$

A2 Proof of Proposition 1

It is useful to define the second order approximation of the cost of funds R_j^{-x} with respect to a shock to the interest rate of bank b, where x is an arbitrary positive number.

Up to second order

$$R_j^{-x} \approx \bar{R}_j^{-x} + \frac{dR_j^{-x}}{d\ln R_b}u + \frac{1}{2}\frac{d^2R_j^{-x}}{d\ln R_b^2}u^2 \tag{41}$$

First, we differentiate R_j^{-x}:

$$R_j^{-x} = \left(\psi_B(\sum_{k\in B} T_{jk}R_k^{-\theta})^{\frac{\varphi}{\theta}} + (1-\psi_B)R_{jM}^{-\varphi}\right)^{(-\frac{1}{\varphi})(-x)}$$

Letting $\Phi = \psi_B(\sum_{k\in B} T_{jk}R_k^{-\theta})^{\frac{\varphi}{\theta}} + (1-\psi_B)R_{jM}^{-\varphi}$, we have:

$$\frac{dR_j^{-x}}{d\ln R_b} = -x\Phi^{(-\frac{1}{\varphi})(-x)-1}\psi_B\Big(\sum_{k\in B} T_{jk}R_k^{-\theta}\Big)^{\frac{\varphi}{\theta}-1}T_{jb}R_b^{-\theta}$$

Replacing $T_{jb}R_b^{-\theta} = v_{jb}\Big(\sum_{k\in B} T_{jk}R_k^{-\theta}\Big)$ and re-arranging yields:

$$\frac{dR_j^{-x}}{d\ln R_b} = -x\Phi^{(-\frac{1}{\varphi})(-x)-1}\psi_B\Big(\sum_{k\in B} T_{jk}R_k^{-\theta}\Big)^{\frac{\varphi}{\theta}}v_{jb}$$

Then we can use the definition of s_j which implies that $\Phi^{-1} = \frac{s_j}{\psi_B R_{jB}^{-\varphi}}$:

$$\frac{dR_j^{-x}}{d\ln R_b} = -x\Phi^{(-\frac{1}{\varphi})(-x)}\frac{s_j}{R_{jB}^{-\varphi}}\Big(\sum_{k\in B} T_{jk}R_k^{-\theta}\Big)^{\frac{\varphi}{\theta}}v_{jb}$$

Now, using the definition of R_{jB} we have that $R_{jB}^{-\theta} = \Big(\sum_{k\in B} T_{jk}R_k^{-\theta}\Big)$, which implies:

$$\frac{dR_j^{-x}}{d\ln R_b} = -x\Phi^{(-\frac{1}{\varphi})(-x)}\frac{s_j}{R_{jB}^{-\varphi}}R_{jB}^{-\varphi}v_{jb}$$

Lastly, replacing Φ, we get:

$$\frac{dR_j^{-x}}{d\ln R_b} = -xR_j^{-x}s_jv_{jb} \tag{42}$$

We can express the second derivative as:

$$\frac{d^2 R_j^{-x}}{d\ln R_b^2} = x^2 R_j^{-x} v_{jb}^2 s_j^2 - x R_j^{-x} s_j \frac{dv_{jb}}{d\ln R_b} - x R_j^{-x} v_{jb} \frac{ds_j}{d\ln R_b}$$

where $\frac{dv_{jb}}{d\ln R_b} = -\theta v_{jb}(1-v_{jb})$ and $\frac{ds_j}{d\ln R_b} = -\varphi v_{jb} s_j(1-s_j)$. This can be shown as follows. First, we have that:

$$\frac{dv_{jb}}{d\ln R_b} = -\theta\left(\frac{T_{jb}R_b^{-\theta}}{\sum_{k\in B} T_{jk}R_k^{-\theta}} - \frac{(T_{jb}R_b^{-\theta})^2}{(\sum_{k\in B} T_{jk}R_k^{-\theta})^2}\right)$$

Expanding the expression for $-\theta v_{jb}(1-v_{jb})$ yields:

$$-\theta v_{jb}(1-v_{jb}) = -\theta\left[\frac{T_{jb}R_b^{-\theta}\sum_{k\in B} T_{jk}R_k^{-\theta} + (T_{jb}R_b^{-\theta})^2}{(\sum_{k\in B} T_{jk}R_k^{-\theta})^2}\right].$$

A similar argument shows that $\frac{ds_j}{d\ln R_b} = -\varphi v_{jb} s_j(1-s_j)$.

Combining all terms we have:

$$\frac{d^2 R_j^{-x}}{d\ln R_b^2} = x^2 R_j^{-x} v_{jb}^2 s_j^2 + x R_j^{-x} s_j \theta v_{jb}(1-v_{jb}) + x R_j^{-x} v_{jb}^2 \varphi s_j(1-s_j) \tag{43}$$

Now, we can use 42 and 43 in 41:

$$R_j^{-x} \approx \bar{R}_j^{-x} - x R_j^{-x} s_j v_{jb} u + \frac{1}{2}\left(x^2 R_j^{-x} v_{jb}^2 s_j^2 + x R_j^{-x} s_j \theta v_{jb}(1-v_{jb}) + x R_j^{-x} v_{jb}^2 \varphi s_j(1-s_j)\right)u^2$$

or, re-arranging and evaluating at the point around which the approximation is being taken:

$$R_j^{-x} \approx \bar{R}_j^{-x}\left(1 - x\bar{s}\bar{v}_{jb}u + x^2\bar{v}_{jb}^2\bar{s}^2\frac{u^2}{2} + x\bar{s}\theta\bar{v}_{jb}(1-\bar{v}_{jb})\frac{u^2}{2} + x\bar{v}_{jb}^2\varphi\bar{s}(1-\bar{s})\frac{u^2}{2}\right) \tag{44}$$

Applying expectations across firms, we can find an expression for $\mathbb{E}(R_j^{-x})$:

$$\mathbb{E}R_j^{-x} \approx \bar{R}_j^{-x}\left(1 - x\bar{s}\mathbb{E}(\bar{v}_{jb})u + x^2\mathbb{E}(\bar{v}_{jb}^2)\bar{s}^2\frac{u^2}{2} + x\bar{s}\theta\mathbb{E}(\bar{v}_{jb})(1-\mathbb{E}(\bar{v}_{jb}))\frac{u^2}{2} + x\mathbb{E}(\bar{v}_{jb}^2)\varphi\bar{s}(1-\bar{s})\frac{u^2}{2}\right)$$

Next, applying logs and differencing the initial value (for sufficiently small u):

$$\Delta\mathbb{E}R_j^{-x} \approx -x\bar{s}\mathbb{E}(\bar{v}_{jb})u + x^2\mathbb{E}(\bar{v}_{jb}^2)\bar{s}^2\frac{u^2}{2} + x\bar{s}\theta\mathbb{E}(\bar{v}_{jb})(1-\mathbb{E}(\bar{v}_{jb}))\frac{u^2}{2} + x\mathbb{E}(\bar{v}_{jb}^2)\varphi\bar{s}(1-\bar{s})\frac{u^2}{2}$$

Lastly, renaming $\mathbb{E}(\bar{v}_{jb})) = \mu_b$ and $\mathbb{E}(\bar{v}_{jb}^2) = \sigma_b^2 + \mu_b^2$, we get:

$$\Delta\mathbb{E}R_j^{-x} \approx -x\bar{s}\mu_b u + x^2(\sigma_b + \mu_b^2)\bar{s}^2\frac{u^2}{2} + x\bar{s}\theta(\mu_b - \sigma_b^2 - \mu_b^2)\frac{u^2}{2} + x(\sigma_b + \mu_b^2)\varphi\bar{s}(1-\bar{s})\frac{u^2}{2} \tag{45}$$

Now we can express the second order approximation of log aggregate output in terms of the expression in 45, as follows:

$$\Delta \log Y \approx \frac{1+\eta\phi}{\phi(\eta-1)}\Delta \mathbb{E}R_j^{-x_1} + \frac{1-\alpha\phi}{\phi(\alpha+1)}\Delta \mathbb{E}R_j^{-x_2}$$

where $x_1 = \frac{(\eta-1)\alpha}{\alpha+\eta}$ and $x_2 = \frac{\eta(\alpha+1)}{\alpha+\eta}$. I will denote with A the term $\frac{1+\eta\phi}{\phi(\eta-1)}\Delta \mathbb{E}R_j^{-x_1}$ and with B the term $\frac{1-\alpha\phi}{\phi(\alpha+1)}\Delta \mathbb{E}R_j^{-x_2}$. Expanding A we get:

$$\begin{aligned} A = &-\frac{(1+\eta\phi)\alpha}{\phi(\alpha+\eta)}\bar{s}\mu_b u + \frac{(1+\eta\phi)(\eta-1)\alpha^2}{\phi(\alpha+\eta)^2}\bar{s}(\sigma_b+\mu_b^2)\frac{u^2}{2} \\ &+ \frac{(1+\eta\phi)\alpha}{\phi(\alpha+\eta)}\theta\bar{s}(\mu_b-\sigma_b^2-\mu_b^2)\frac{u^2}{2} + \frac{(1+\eta\phi)\alpha}{\phi(\alpha+\eta)}\varphi(\sigma_b+\mu_b^2)\bar{s}(1-\bar{s})\frac{u^2}{2} \end{aligned}$$

Doing the same with B yields

$$\begin{aligned} B = &-\frac{(1-\alpha\phi)\eta}{\phi(\alpha+\eta}\bar{s}\mu_b u + \frac{(1-\alpha\phi)(\alpha+1)\eta^2}{\phi(\alpha+\eta)^2}(\sigma_b+\mu_b^2)\bar{s}^2\frac{u^2}{2} \\ &+ \frac{(1-\alpha\phi)\eta}{\phi(\alpha+\eta)}\theta\bar{s}(\mu_b-\sigma_b^2-\mu_b^2)\frac{u^2}{2} + \frac{(1-\alpha\phi)\eta}{\phi(\alpha+\eta)}\varphi\bar{s}(1-\bar{s})(\sigma_b+\mu_b^2)\frac{u^2}{2} \end{aligned}$$

Then I will use subscripts to denote each term in expressions A and B. Adding the first terms of A and B:

$$A_1 + B_1 = -\frac{1}{\phi}\bar{s}\mu_b u$$

Adding A_2 and B_2:

$$A_2 + B_2 = \frac{1}{\phi}\Big[\frac{\alpha\eta(1-\phi)-\alpha+\eta}{(\alpha+\eta)}\Big](\sigma_b+\mu_b^2)\bar{s}^2\frac{u^2}{2}$$

Adding A_3 and B_3 yields:

$$A_3 + B_3 = \frac{1}{\phi}\bar{s}\theta(\mu_b-\sigma_b^2-\mu_b^2)\frac{u^2}{2}$$

Doing the same for A_4 and B_4:

$$A_4 + B_4 = \frac{1}{\phi}(\sigma_b+\mu_b^2)\varphi\bar{s}(1-\bar{s})\frac{u^2}{2}$$

Putting everything together

$$\Delta \log Y \approx -\frac{1}{\phi}\Bigg(\bar{s}\mu_b u - \Big[\frac{\alpha\eta(1-\phi)-\alpha+\eta}{(\alpha+\eta)}\Big](\sigma_b+\mu_b^2)\bar{s}^2\frac{u^2}{2} - \bar{s}\theta(\mu_b-\sigma_b^2-\mu_b^2)\frac{u^2}{2} - (\sigma_b+\mu_b^2)\varphi\bar{s}(1-\bar{s})\frac{u^2}{2}\Bigg)$$

Letting $\Omega = \frac{\alpha\eta(1-\phi)-\alpha+\eta}{(\alpha+\eta)}$

$$\Delta \log Y \approx -\frac{1}{\phi}\Bigg(\bar{s}\mu_b u - \Omega\bar{s}^2(\sigma_b^2+\mu_b^2)\frac{u^2}{2} - \theta\bar{s}(\mu_b-\sigma_b^2-\mu_b^2)\frac{u^2}{2} - \varphi\bar{s}(1-\bar{s})(\sigma_b^2+\mu_b^2)\frac{u^2}{2}\Bigg) \tag{46}$$

A3. Proof of Proposition 2

The lending rate R_B increases by u since the lending rates of each individual bank increase by u. We can approximate the change in the lending rate of firm j as follows:

$$R_j^{-x} \approx \bar{R}_j^{-x} + \frac{dR_j^{-x}}{d\ln R_B}u + \frac{1}{2}\frac{d^2R_j^{-x}}{d\ln R_B^2}u^2 \tag{47}$$

We have

$$\frac{dR_j^{-x}}{d\log R_B} = -xs_jR_j^{-x}$$

$$\begin{aligned}\frac{dR_j^{-x2}}{d\log R_B^2} &= x^2s_jR_j^{-x} - xR_j^{-x}\frac{\partial s_j}{\partial \log R_B}\\ &= x^2R_j^{-x}s_j^2 + xR_j^{-x}\varphi s_j(1-s_j)\end{aligned}$$

Using these expressions, taking logs and differencing the initial point yields:

$$\Delta R_j^{-x} \approx -x\bar{s}u + x^2\bar{s}^2\frac{u^2}{2} + x\varphi\bar{s}(1-\bar{s})\frac{u^2}{2}$$

Replacing x for the exponents in the definition of log output yields:

$$\Delta \log Y = -\frac{1}{\phi}\left(\bar{s}u - \Omega\bar{s}^2\frac{u^2}{2} - \varphi\bar{s}(1-\bar{s})\frac{u^2}{2}\right) \tag{48}$$

where $\Omega = \frac{\alpha\eta(1-\phi)-\alpha+\eta}{(\alpha+\eta)}$ as before.

Proof of Proposition 3

Firm-level output is given by

$$Y_j = \left(\frac{\eta}{\eta-1}\right)^{-\frac{\eta\alpha}{\alpha+\eta}} \quad Y^{\frac{\alpha}{\alpha+\eta}} \quad z_j^{\eta\frac{\alpha+1}{\alpha+\eta}} \quad w^{-\eta\frac{\alpha}{\alpha+\eta}} \quad R_j^{-\eta\frac{\alpha}{\alpha+\eta}} \quad L^{\frac{\eta}{\alpha+\eta}}$$

Taking logs we get

$$\log Y_j = -\frac{\eta\alpha}{\alpha+\eta}\log\left(\frac{\eta}{\eta-1}\right) + \frac{\alpha}{\alpha+\eta}\log Y + \eta\frac{\alpha+1}{\alpha+\eta}\log z_j - \eta\frac{\alpha}{\alpha+\eta}w - \eta\frac{\alpha}{\alpha+\eta}\log R_j + \frac{\eta}{\alpha+\eta}\log L$$

I will collapse the first, second, fourth and fifth terms into a single term called $\log\Theta_t$, which is common to all the firms, and will therefore become irrelevant in computing the result.

$$\log Y_j = \log \Theta_t + \eta \frac{\alpha+1}{\alpha+\eta} \log z_j - \eta \frac{\alpha}{\alpha+\eta} \log R_j$$

Taking temporal differences we get:

$$\Delta \log Y_j = \Delta \log \Theta_t + \eta \frac{\alpha+1}{\alpha+\eta} \Delta \log z_j - \eta \frac{\alpha}{\alpha+\eta} \Delta \log R_j$$

A second-order Taylor expansion of $\log R_j$ that coincides with assumption 2 yields:

$$\log R_j \approx \log \bar{R}_j + \psi_B \bar{\nu}_{jb} u + \bar{\nu}_{jb}^2 \psi_B^2 \frac{u^2}{2} - \psi_B \theta \bar{\nu}_{jb}(1-\bar{\nu}_{jb}) \frac{u^2}{2} - \bar{\nu}_{jb}^2 \varphi \psi_B (1-\bar{s}) \frac{u^2}{2} \tag{49}$$

This expression follows from setting $x = -1$ in equation 44 and taking logs. Next, differentiating with respect to the initial point and using the fact that at the initial point $\nu_{jb} = T_{jb}$, we get:

$$\Delta \log R_j \approx \psi_B T_{jb} u + \psi_B^2 T_{jb}^2 \frac{u^2}{2} - \theta \psi_B T_{jb}(1-T_{jb}) \frac{u^2}{2} - \varphi \psi_B (1-\bar{s}) T_{jb}^2 \frac{u^2}{2} \tag{50}$$

Plugging this expression into equation 49 yields a second order approximation of firm level output after one bank suffers an increase in its lending terms.

$$\Delta \log Y_j = \Delta \log \Theta_t + \eta \frac{\alpha+1}{\alpha+\eta} \Delta \log z_j - \eta \frac{\alpha}{\alpha+\eta} \left(\psi_B T_{jb} u + \psi_B^2 T_{jb}^2 \frac{u^2}{2} - \theta \psi_B T_{jb}(1-T_{jb}) \frac{u^2}{2} - \varphi \psi_B (1-\bar{s}) T_{jb}^2 \right.$$

The cross-sectional regression of log output changes on pre-existing exposure can be computed by a simple regression estimated by OLS:

$$\Delta \log Y_j = \beta_0 + \beta_{output} T_{jb} + \epsilon_f$$

In this setting the exposure in the pre-period to the affected bank is just T_{jb}. The regression coefficient is given by the covariance between $\log Y_j$ and T_{jb}. Since all the firms have the same $\Delta \log \Theta_t$ regardless of their specific T_{jb}, then the effect on aggregates of the shock is absorbed by the intercept. The regression coefficient in the population is given by:

$$\beta_{output} = \frac{cov(\Delta \log Y_j, T_{jb})}{var(T_{jb})}$$

The covariance between $\log Y_j$ and T_{jb} is given by:

$$cov(\Delta \log Y_j, T_{jb}) = \eta \frac{\alpha}{\alpha+\eta} \bar{s} u \Bigg(cov(T_{jb}, T_{jb}) + \psi_B \, cov(T_{jb}, T_{jb}^2) \frac{u}{2}$$
$$- \theta \, cov(T_{jb}, T_{jb}(1-T_{jb})) \frac{u}{2} - \varphi(1-\psi_B) \, cov(T_{jb}, T_{jb}^2) \frac{u}{2} \Bigg)$$

where we use the fact that $cov(T_{jb}, \Delta \log \Theta_t) = 0$ and $cov(T_{jb}, \Delta \log z_t) = 0$. Dividing by

$var(T_{jb})$ and expanding the term $cov(T_{jb}, T_{jb}(1-T_{jb}))$, we get:

$$\beta_{output} = \frac{cov(\Delta \log Y_j, T_{jb})}{var(T_{jb})}$$

$$= \eta \frac{\alpha}{\alpha+\eta} \psi_B u \left(1 + \psi_B \frac{u}{2} \frac{cov(T_{jb}, T_{jb}^2)}{var(T_{jb})} - \theta \frac{u}{2} \left(1 - \frac{cov(T_{jb}, T_{jb}^2)}{var(T_{jb})} \right) - \varphi(1-\psi_B) \frac{u}{2} \frac{cov(T_{jb}, T_{jb}^2)}{var(T_{jb})} \right)$$

This is the main result.

Proof of Proposition 4

Firm-level credit in logs can be written as

$$\log Q_j = \log \Sigma + \log s_j + \frac{1+\alpha}{\alpha} \Big(\log Y_j - \log z_j \Big)$$

for a variable Σ that, similar to the previous proof, captures every term that affects firms regardless of their treatment.

Therefore, an OLS regression

$$\Delta \log Q_j = \beta_0 + \beta_{credit} T_{jb} + \zeta_j$$

can be expressed simply as : $\beta_{credit} = \beta_{share} + \frac{1+\alpha}{\alpha} \beta_{output}$ where $\beta_{share} = \frac{cov(\Delta \log s_j, T_{jb})}{var(T_{jb})}$.

Second order approximation of $\log s_j$**.** We know that

$$s_j = \frac{\psi_B R_{jB}^{-\varphi}}{\psi_B R_{jB}^{-\varphi} + (1+\psi_B) R_{jM}^{-\varphi}} \qquad R_{jB} = \Big(\sum_{b \in B} T_{jb} R_b^{-\theta} \Big)^{-\frac{1}{\theta}}$$

Differentiating $\log s_j$ with respect to $\log R_b$ yields

$$\frac{d \log s_j}{d \log R_b} = -\varphi v_{jb} + \varphi s_j v_{jb}$$

The second derivative can then be expressed as

$$\frac{d^2 \log s_j}{d \log R_b^2} = -\varphi \frac{\partial v_{jb}}{\partial \log R_b} + \varphi v_{jb} \frac{\partial s_j}{\partial \log R_b} + \varphi s_j \frac{\partial v_{jb}}{\partial \log R_b}$$

We can use the expressions for $\frac{\partial v_{jb}}{\partial \log R_b}$ and $\frac{\partial s_j}{\partial \log R_b}$ from previous proofs. Replacing, yields:

$$\frac{d^2 \log s_j}{d \log R_b^2} = \theta \varphi v_{jb}(1-v_{jb}) + \varphi^2 v_{jb}^2 s_j (1-s_j) - \varphi \theta s_j v_{jb} (1-v_{jb})$$

The second order approximation of $\log s_j$ around $log \bar{s}$ can be expressed as

$$\log s_j \approx \log \bar{s} + \frac{d \log s_j}{d \log R_b} u + \frac{d^2 \log s_j}{d \log R_b^2} \frac{u^2}{2}$$

Plugging in our expressions for the first and second derivatives, yields

$$\log s_j \approx \log \bar{s} - \varphi(1-\bar{s})T_{jb}u + \Big(\theta\varphi T_{jb}(1-T_{jb}) + \varphi^2 T_{jb}^2 \bar{s}(1-\bar{s}) - \varphi\theta\bar{s}T_{jb}(1-T_{jb})\Big)\frac{u^2}{2}$$

Substracting the intial point,

$$\begin{aligned}
\Delta \log s_j &\approx -\varphi(1-\bar{s})T_{jb}u + \Big(\theta\varphi T_{jb}(1-T_{jb}) + \varphi^2 T_{jb}^2 \bar{s}(1-\bar{s}) - \varphi\theta\bar{s}T_{jb}(1-T_{jb})\Big)\frac{u^2}{2} \\
&\approx -\varphi\left((1-\bar{s})T_{jb}u - \theta T_{jb}(1-T_{jb})\frac{u^2}{2} + \varphi T_{jb}^2\bar{s}(1-\bar{s})\frac{u^2}{2} + \bar{s}\theta T_{jb}(1-T_{jb})\frac{u^2}{2}\right) \\
&\approx -\varphi\left((1-\bar{s})T_{jb}u - \theta(1-\bar{s})T_{jb}(1-T_{jb})\frac{u^2}{2} + \varphi T_{jb}^2\bar{s}(1-\bar{s})\frac{u^2}{2}\right)
\end{aligned}$$

The expression for β_{credit} using the expression for $\Delta \log s_j$ is:

$$\beta_{credit} = \frac{1+\alpha}{\alpha}\beta_{output} - \varphi(1-\bar{s})u\left(1 + \varphi\bar{s}\frac{u}{2}\frac{cov(T_{jb}, T_{jb}^2)}{var(T_{jb})} - \theta\frac{u}{2}\Big(1 - \frac{cov(T_{jb}, T_{jb}^2)}{var(T_{jb})}\Big)\right) \quad (51)$$